

Leveraging mathematical methods for environmentally friendly waste management: Locating optimal facility sites

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Abstract: This study examines the Uncapacitated Facility Location Problem (UFLP) in the context of landfill placement, a critical factor in sustainable waste management. It introduces an enhanced model that integrates operational logistics, cost structures, demand distribution, temporal dynamics, and population growth projections to optimize landfill siting strategies. We apply the proposed model in a case study of Cape Coast Metropolis, Ghana, which includes twenty-three suburbs and seven potential landfill sites. The analysis evaluates the impact of landfill distribution on waste management efficiency and operational expenses over time. Results indicate that while increasing the number of landfills improves waste distribution, it also escalates long-term operational costs. The study underscores the need for strategic planning to balance efficiency and cost-effectiveness. Additionally, incorporating temporal dynamics is crucial for long-term sustainability. The study highlights the importance of integrating economic, operational, and environmental considerations in landfill siting decisions, providing a foundation for future research on sustainable waste management. Optimized landfill placement can lead to significant cost savings and improved resource allocation. The findings inform policymakers and businesses in developing regulations that promote efficient facility placement, enhance disaster response, and support long-term waste management sustainability.

Keywords: Accessibility, Optimal locations, Semi-obnoxious facilities, Sustainability, Transportation costs, Waste collection, Waste disposal.

1. Introduction

Facilities are fundamental to shaping our lives and environments, from libraries that enhance social well-being to factories that drive economic growth [1, 2]. The selection and placement of these facilities are critical, influencing both quality of life and environmental sustainability. While desirable facilities, such as parks, positively contribute to our surroundings, undesirable ones, like landfills, can present significant challenges [3].

Facility location optimisation is critical to operational efficiency across various sectors, from logistics and transportation [4, 5] to waste management and healthcare services. This study area has gathered significant attention over the past decade, with researchers focusing on developing models that balance cost-effectiveness with service quality. One of the most prominent models in this domain is the Uncapacitated Facility Location Problem [6] which seeks to determine the optimal locations for facilities to minimise operational costs while meeting demand. The UFLP, though effective in various applications, has inherent limitations, particularly in its static nature and the assumption of infinite facility capacity. Recent studies have addressed these limitations by introducing dynamic variables and considering factors such as demand variability and environmental impact [7-9].

The UFLP has traditionally focused on minimising transportation and facility setup expenses, often overlooking the dynamic nature of demand points over time. For instance, studies by Atta, et al. [10] and Srivastava and Jha [7] have highlighted the significance of incorporating temporal dynamics into facility location models to predict better and accommodate future demand. While these studies have made considerable strides in improving the UFLP's applicability, they often need to fully integrate population growth rates and their impact on waste generation and resource allocation. The challenge lies in the model's ability to adapt to these changes while maintaining cost efficiency and service quality, an issue that has been partially addressed but has yet to be entirely resolved in recent research [11, 12].

One of the significant strengths of recent advancements in facility location models is their incorporation of multi-objective optimisation techniques, which allow for a more balanced approach to resource distribution. For example, Ghadge, et al. [13] and Pang and Zhou [14] have demonstrated the potential of hybrid models that integrate economic and environmental objectives, offering more sustainable solutions in facility location planning. However, these models still need to improve scalability and handle large datasets effectively, particularly in urban planning scenarios where numerous demand points are highly variable. Additionally, while these models consider environmental impacts, they often fall short when it comes to addressing long-term sustainability goals, particularly in waste management, where population growth and waste generation rates are critical factors [15, 16].

Our research builds on a robust body of literature on spatial facility location problems [17, 18] with significant foundational work laid by Church and Murray [19]. Previous research in spatial optimisation, including methodologies for the UFLP, has provided substantial insights into the complexities of facility placement [20, 21]. This work has significantly enhanced our understanding of the challenges that are likely to arise, particularly in sustainable waste management [22, 23]. Building on these established concepts, we introduce a novel mathematical model that improves the traditional UFLP framework [6] by incorporating dynamic factors, including population growth rates and future demand projections.

Unlike prior approaches which have often overlooked these critical variables, our model optimises landfill placement by balancing short-term operational efficiency with long-term cost-effectiveness while accounting for the environmental impact [24]. This advancement seeks to address the gaps identified in existing UFLP models, which typically need to account for changes in population over extended periods. To validate our model, we conducted a focused case study in Cape Coast Metropolis, Ghana, examining twenty-three distinct suburbs and seven potential landfill sites. This case study provides valuable insights into the interactions between facility placement, resource distribution, and long-term cost efficiency, allowing for a comprehensive analysis of trade-offs between facility count, travel distances, and operational costs [25].

By addressing these challenges, our research aims to provide a significant contribution to the field by offering a robust and adaptable framework for optimising landfill placement [26]. Our model is poised to guide policy decisions regarding future waste management infrastructure needs, optimising resource allocation [26] while minimising environmental impact. As sustainable waste management remains a critical global issue for urban centres, this study introduces a novel approach that can substantially improve landfill placement strategies, significantly contributing to theory and practise. The novel contribution of the current study lies in its development of a Modified Uncapacitated Facility Location Problem (MUFLP) model that explicitly integrates population growth rates and temporal dynamics into the decision-making process. It is undeniable and inevitable that waste is generated wherever there is a human presence [27]. With the expansion of human populations and the rise of affluent societies, waste production concurrently increases over time [28]. Population growth and the passage of time are notable factors that contribute significantly to waste generation [29]. A study by Srivastava and Jha [7] in Prayagraj, India, revealed a significant correlation between population growth, employment, household size, and waste generation rates. This correlation underscores the critical role the population growth plays in the gradual accumulation of waste over time [30].

Unlike traditional models which assume that the demand points are static, the MUFLP model projects future waste generation at each demand point, allowing for more accurate and sustainable facility placement over time. This approach addresses the gaps identified in previous studies and enhances the model's applicability in real-world scenarios where demand is expected to grow and evolve. By incorporating these dynamic factors, the MUFLP model provides a more robust framework for long-term facility location planning, ensuring that facilities are optimally positioned to meet current and future demand while minimising costs and the environmental impact [31]. This advancement marks a significant leap in facility location optimisation, providing a more comprehensive solution to the challenges highlighted in the existing literature.

The remainder of the manuscript is structured as follows: Section 2 provides an overview of previous research on the UFLP model, focusing on valuable contributions to the field. Section 3 deals with the research design and methodology while Section 4 deals with the materials and methods employed in the study. Section 5 presents the outcomes derived from the MATLAB implementations. Section 6 offers a thorough analysis and discussion of the simulations, providing valuable insights into the model's performance and confirming the validity of our findings. Section 7 explores the implications of implementing the UFLP models in this context. Finally, Section 8 summarises the study's main findings, while Section 9 offers observations and recommendations to improve the model's effectiveness in various geographical contexts.

2. Literature Review

Numerous models and algorithms have been effectively employed to address the complexities of facility location within waste management, as evidenced by the existing literature. These studies often prioritise minimising transportation costs, facility setup expenses, and other relevant factors [15]. For example, Pires, et al. [16] proposed a linear programming framework for single and multi-objective problems in sustainable waste management, focusing on challenges like vehicle routing and transportation. Similarly, Olapiriyakul, et al. [9] introduced an optimisation model for waste management network design, integrating environmental and social impact measures with economic objectives.

Ghadge, et al. [13] emphasised the importance of finding sustainable facility location solutions for a closed-loop distribution network, particularly in the uncertain environment of online retailing, where there is an increasing need to reduce carbon emissions. Their study uses a case study approach to improve distribution centre location decisions for single and double hub scenarios. It proposes a hybrid approach that combines the centre of gravity method with mixed-integer programming [32]. The model, validated with empirical data from a major UK retail distributor network, suggests adopting a two-hub facility location strategy to mitigate emerging risks and disruptions in the supply chain [33].

Despite the advancements in this research area, the existing approaches have neglected to consider population growth rate and time as factors influencing waste generation. This omission presents a significant challenge. As Srivastava and Jha [7] highlighted, population growth directly correlates with increased waste generation. As emphasised in the work of Adeleke and Olukanni [12] ignoring this dynamic can lead to inadequate waste management infrastructure, as improper facility placement due to underestimation of future waste volumes becomes a significant issue. Additionally, waste collection routes, according to De Armas, et al. [34] may become inefficient as populations grow and waste generation patterns shift and ultimately, these issues can increase operational costs [35].

To address this gap, we propose a multi-step approach. The first step involves leveraging existing Uncapacitated Facility Location Problem (UFLP) models, such as those presented by Atta, et al. [36] and Zhang, et al. [37] and using real-world data to identify optimal facility locations based on proximity to population centres. This initial step provides valuable insights into resource allocation and service optimisation. However, recognising the limitations of the existing UFLP model - particularly its oversight of the population growth rate and temporal dynamics - necessitates a refinement phase. This phase involves adapting the UFLP model to incorporate variables and constraints that explicitly address

these overlooked dimensions, specifically integrating population growth rate projections and temporal considerations into the model's framework, and builds upon the work of Ortiz-Astorquiza, et al. [38] who explored multilevel Uncapacitated p -location problems.

The proposed adjustments promise to enhance the model's predictive accuracy, allowing for more precise estimations of waste generation volumes over time [39] which will enable stakeholders to anticipate future waste management needs and allocate resources accordingly. Moreover, by factoring in population growth rate and temporal dynamics, the refined model facilitates proactive decision-making, reducing the likelihood of underutilised or inadequately positioned facilities, as encountered in the study by Asgari, et al. [11]. Furthermore, these modifications are expected to yield broader benefits beyond operational enhancements. By accommodating population growth rate and temporal considerations, the refined UFLP model can optimise cost-effectiveness, bolster service quality [35] and mitigate ecological concerns. The enhanced predictive capabilities of the model empower decision-makers to proactively address environmental sustainability goals, aligning waste management strategies with broader ecological imperatives.

Empirical applications demonstrate the effectiveness of spatial optimisation models enabled by Geographic Information Systems (GIS) technology. These models help minimise representation errors and better align with real-world demand distributions.

This integration offers a robust framework for decision-makers in facility location planning, addressing the difficulties of accurately representing continuous demand [40].

Additionally, Torkayesh, et al. [41] developed an integrated decision-making model for selecting disposal locations for medical waste. Their model incorporates GIS, the Best-Worst technique, and the Measurement of Alternatives and Ranking Using the Compromise Solution (MARCOS) technique. The Best-Worst technique is used to build GIS suitability maps, which analyse location criteria, while the MARCOS approach prioritises eight prospective dump sites, considering environmental factors.

A comprehensive examination of the literature reveals that various models and algorithms have successfully tackled the complexities of facility location problems. Historically, the primary focus when solving Uncapacitated Facility Location Problems (UFLPs) has been minimising transportation costs, facility setup expenses, and other relevant factors, which are indeed crucial [15].

The existing UFLP model, as discussed by De Armas, et al. [34] overlooks two crucial variables: population growth rate and temporal dynamics. These factors must be considered to avoid complications, particularly when managing a growing customer base. As the customer base expands, it becomes increasingly important to develop distinct facility layouts and service strategies to address challenges such as improper facility placement, inefficient service delivery, and rising expenses [42]. Therefore, incorporating these variables into the UFLP model could lead to more accurate predictions and efficient waste management strategies.

A multifaceted approach is necessary to bridge this gap in waste management optimisation. The first step involves applying the established UFLP model to empirical data derived from real-world contexts. This empirical grounding facilitates the identification of optimal facility locations by assessing their proximity to settlements and population centres.

3. Research Design and Methodology

3.1. Research Design

This study employs quantitative research design, utilising mathematical modelling and computational analysis to address the UFLP in the context of sustainable waste management. The research is structured to build existing models while incorporating new variables, such as population growth and temporal dynamics, which are critical for long-term planning in waste management. This approach aligns with the current trends in facility location research, where integrating dynamic variables is increasingly recognised as essential for optimising resource allocation in changing environments [43, 44].

3.2. Research Questions

The following research questions will guide our study:

1. How can the traditional UFLP model be enhanced to better accommodate the dynamic nature of waste generation, which is influenced by population growth and temporal factors?
2. What is the optimal configuration of landfill sites in the Cape Coast Metropolis, Ghana, that balances cost-efficiency with sustainability over a 50-year planning horizon?
3. How does the inclusion of population growth rates and temporal dynamics impact the effectiveness and cost-efficiency of waste management facilities?

These questions are designed to explore the practical applications of advanced mathematical models with regard to optimising waste facility locations and to evaluate the long-term sustainability of these solutions. Such inquiries reflect the growing consensus in the literature that facility location models must evolve to address the complexities of real-world applications, particularly in the context of sustainability [7, 8].

3.3. Hypotheses

Based on the research questions, the study tests the following hypotheses:

1. H1: Including population growth rates and temporal dynamics in the UFLP model will result in more optimal landfill placements than the traditional UFLP model.
2. H2: The Modified Uncapacitated Facility Location Problem (MUFLP) model will demonstrate greater cost-efficiency over time by reducing the need for facility reallocation and minimising operational costs.
3. H3: Considering future demand growth, a spatially optimised configuration of landfill sites will lead to a more balanced and sustainable distribution of waste management resources across the Cape Coast Metropolis.

These hypotheses are designed to test the proposed model's effectiveness and applicability in real-world waste management scenarios. The focus on cost-efficiency and sustainability is consistent with previous research, which emphasises the importance of these factors in facility location optimization [15, 16].

3.4. Methodology

The methodology for this study involves several key steps:

Model Development:

A Modified Uncapacitated Facility Location Problem (MUFLP) model is developed, incorporating population growth projections and temporal dynamics. This model is based on the traditional UFLP framework but enhanced to address the specific challenges of sustainable waste management. This approach builds on the work of Yao and Murray [18] who have emphasised the need for models that can adapt to future changes in demand.

3.4.1. Data Collection:

Data on population growth rates, waste generation patterns, and geographic information from the Cape Coast Metropolis are collected, including historical data and projections over a 50-year period, as provided by the Ghana Statistical Service and other relevant sources. Recent studies have highlighted the importance of accurate and comprehensive data in facility location modelling, underscoring the need for reliable inputs to achieve robust model outcomes [9].

3.4.2. Computational Analysis:

The MUFLP model is implemented using MATLAB, in which various scenarios are simulated to determine the optimal number and locations of landfill sites. The model's performance is evaluated

against the baseline UFLP model using key performance indicators such as cost minimisation, travel time reduction, and adaptability to changing conditions. This computational approach is consistent with the methods used in contemporary facility location research, which often relies on advanced software tools for simulation and optimization [6].

3.4.3. Validation and Comparison:

The results from the MUFLP model are compared with those of the traditional UFLP model. The comparison focuses on each model's cost-effectiveness, operational efficiency, and long-term sustainability, validating the hypotheses outlined earlier. This comparative analysis is crucial for understanding the relative advantages of different modelling approaches, as discussed in the literature by Zhang, et al. [37].

3.4.3. Sensitivity Analysis:

A sensitivity analysis assesses the model's robustness under different population growth and waste generation assumptions. This analysis helps identify potential risks and uncertainties in the model's predictions. Sensitivity analysis in facility location research is well-documented, particularly with regard to its role in enhancing the reliability and applicability of model outcomes [40, 44].

4. Materials and Methods

This study adopts a comprehensive and systematic approach to improving waste management practises in the Cape Coast Metropolis (CCM). The methodology is designed to address the complex challenges associated with waste management, focusing on optimising facility location, enhancing waste collection efficiency, and promoting active community involvement. The following subsections outline the methods and strategies employed to achieve these objectives.

4.1. Integration of the Improved UFLP Model

A vital component of this research involves the application of an enhanced Uncapacitated Facility Location Problem (UFLP) model, which is referred to as the Modified Uncapacitated Facility Location Problem (MUFLP) model. This model represents a significant advancement over traditional UFLP approaches by incorporating dynamic variables such as population growth projections provided by the Ghana Statistical Service. Accurate predictions are crucial for forecasting future demand points and waste generation volumes across CCM.

The MUFLP model incorporates precise data on demand point locations, enabling a facility placement to be more accurately aligned with current and future waste management needs. Unlike traditional UFLP models, the MUFLP model also considers temporal changes in waste distribution patterns. By accounting for these dynamic factors, the MUFLP model aims to optimise landfill placement in the short and long term, ensuring cost-effectiveness and operational efficiency.

A comparative analysis with a baseline UFLP model will be conducted to evaluate the effectiveness of the MUFLP model. This assessment will focus on key performance indicators (KPIs) such as cost minimisation, travel time reduction, and the model's adaptability to evolving waste generation patterns. The evaluation will provide valuable insights into the strengths and limitations of the MUFLP model, establishing a solid foundation for its practical application in waste management scenarios.

4.2. Spatial Analysis and Optimisation

Efficient waste management requires a deep understanding of the spatial dynamics within CCM. To achieve this, the MUFLP model will be employed to strategically place at least five landfill facilities, ensuring comprehensive coverage of all twenty-three demand points within the metropolis. Strategic placement of waste management resources is crucial for achieving a balanced distribution and minimising operational costs related to waste collection and disposal.

Extensive spatial analysis will use historical waste generation data to identify patterns and trends that can inform facility placement decisions. Additionally, the capabilities of waste collection trucks will be thoroughly assessed, considering factors such as load limits, fuel efficiency, and maintenance schedules. Travel speeds within CCM will also be analysed to determine the optimal distance threshold for placing facilities relative to demand points.

This spatial analysis and optimisation process aims to minimise the distance travelled by waste collection vehicles, thereby reducing fuel consumption, emissions, and operational expenses. The study aims to optimise waste collection operations by strategically locating landfill facilities and promoting a more sustainable waste management system.

4.3. Community Engagement and Education

Community engagement is vital for establishing a foundation of sustainable waste management practises. This study will introduce a range of community engagement and education initiatives to raise awareness about waste reduction, composting, and proper waste segregation practises. These programmes equip residents with the knowledge and tools to actively participate in waste management efforts.

This study will collaborate with local schools, community centres, and non-governmental organisations (NGOs) to disseminate information and organise training sessions. The educational content will be tailored to address the specific needs and challenges of different communities within CCM, focusing on promoting practises that align with the principles of a circular economy.

Furthermore, partnerships with local recycling facilities will be established to create a streamlined process for collecting, sorting, and processing recyclable materials. By integrating these efforts into a comprehensive waste management strategy, this study aims to minimise the amount of waste in landfills and maximise the recovery of valuable resources.

4.4. Evaluation of Social Impact

The primary objective of this study is to ensure equitable distribution of waste management services across all communities within CCM. A thorough social impact assessment (SIA) will be conducted to analyse the potential impacts of waste management practises on various population segments, focusing on marginalised and vulnerable communities.

The SIA will include focus group discussions and stakeholder interviews to gather qualitative data on the social aspects of waste management. These discussions will provide valuable insights into residents' perceptions, concerns, and needs, helping to identify potential disparities in access to waste management services. The results of the SIA will inform the development of plans to address any identified inequities and ensure that all communities benefit fairly from waste management initiatives.

In addition to qualitative methods, quantitative approaches such as surveys and statistical analyses will be employed to assess the social impact of waste management practises. This dual approach will provide a comprehensive understanding of the social implications of facility placement and waste collection strategies, ensuring that the study's recommendations are sustainable from a social and economic perspective.

4.5. Data Security and Management

Given the importance of the data collected during this study, robust data security measures will be implemented to protect participants' information and maintain the integrity of the research process. Advanced security protocols, including cloud-based storage with encryption and k -anonymity techniques, will be utilised to prevent unauthorised access and ensure the confidentiality of sensitive data.

The collected data will include variables such as Euclidean distances between suburbs, projected costs for establishing disposal sites, and demographic information. Each suburb will be assigned a unique MATLAB code to facilitate effective data management and analysis within the computational

framework. This systematic approach will streamline data analysis, allowing for precise conclusions and well-informed recommendations.

Regular audits of data storage systems, secure backups, and strict access controls will be enforced to safeguard sensitive information and restrict access to authorised personnel only.

4.6. Data Collection and Analysis

Data collection will involve a detailed examination of the twenty-three suburbs within CCM, using rigorous criteria to ensure that the collected data are accurate and representative. Key data points will include distances measured along road networks, projected costs for establishing waste disposal sites, and historical waste generation patterns.

MATLAB will be the primary computational tool to process and analyse the spatial data. This software's advanced capabilities allow for intricate data manipulation and visualisation. Models will be created in MATLAB to examine the spatial relationships between demand points and potential landfill sites, enabling the determination of optimal and cost-effective facility locations. The analysis will also consider traffic patterns, road conditions, and population density to ensure the final recommendations are practical and sustainable.

This comprehensive data collection and analysis approach addresses the challenges of representing and understanding spatial relationships in waste management. By integrating spatial analysis with advanced modelling techniques, this study aims to establish a robust framework for optimising facility placement and improving the overall effectiveness of waste management systems within CCM.

Overall, the methodology outlined above represents a comprehensive approach to waste management. It combines advanced modelling, spatial analysis, community engagement, social impact assessment, and meticulous data management practises to ensure that the study's findings are scientifically robust and practically implementable. The goal is to provide valuable insights and recommendations for enhancing waste management practises in the Cape Coast Metropolis.

4.6.1. Analysis of Table 1:

Table 1 lists the settlements in CCM that are designated demand locations for waste management services. A MATLAB code, a label, and the settlement name identify each demand point. Identifying and tagging these demand spots is crucial for future research and modelling stages, which will use spatial optimisation techniques to establish the best locations for waste management facilities.

Table 1.
Considered settlements (demand points).

Demand Points MATLAB Code	Demand Point Label	Demand Point Name
8	A	Brabedzi
9	B	Efutu Mampong
7	C	Efutu
14	D	Brimsu
1	E	Krofofordo
16	F	Nyamoia
19	G	Amoyaw
2	H	Kakumdo
4	I	CCTU
10	J	Akotokyere
11	K	Kwaprow
20	L	Abura
5	M	Nkanfoa
22	N	Third Ridge
15	O	Pedu
23	P	UCC
12	Q	Apewosika
6	R	Adesadel
13	S	Kotokuraba
21	T	Aboom
18	U	Ekon
3	V	Amanful
17	W	OLA

The demand points in the CCM span a large geographic region, highlighting the need for a well-distributed network of waste management facilities. The varieties of the communities involved, ranging from densely populated places like "Kotokuraba" (S) and "UCC" (P) to less congested locations like "Amoyaw" (G) and "Brabedzi" (A), implies that there will be variable waste generation quantities and logistical issues. The spatial distribution of these demand sites emphasises the significance of strategic facility placement to promote fair access and efficiency with regard to trash collection and disposal [15].

By mapping these demand spots, this research can assess each community's proximity to possible facility sites, which is critical for lowering transportation costs and increasing the overall efficiency of the waste management system [16]. Furthermore, identifying the geographical linkages between these demand points enables a more nuanced approach to facility siting, thereby improving the sustainability and cost-effectiveness of waste management operations [9].

Within the geographical area under study, seven potential sites for facilities were identified. These sites are labelled as F1 through F7. The estimated costs for establishing these sites, denoted in thousands of Ghana cedis, are as follows: [70, 65, 67, 62, 70.5, 69, and 73.5], respectively. Furthermore, the direct Euclidean distances between the demand points and the proposed facility sites were determined using Floyd's algorithm, as displayed in Table 2.

Table 2.

The Euclidean distances between pairs of demand points and between demand points and proposed facility sites.

S/No.	Edges	Distance (Meters)
1	(A, B)	905
2	(A, C)	980
3	(A, D)	795
4	(A, F1)	700
5	(B, E)	560
6	(B, F2)	1210
7	(C, F)	1250
8	(D, G)	980
9	(E, H)	1385
10	(E, J)	1705
11	(E, F3)	400
12	(F, G)	1050
13	(F, F1)	600
14	(F, F7)	1000
15	(G, O)	1550
16	(G, N)	1150
17	(G, F7)	1200
18	(H, I)	1685
19	(H, N)	1785
20	(H, F6)	1200
21	(I, L)	1045
22	(J, K)	905
23	(J, T)	1845
24	(J, F4)	980
25	(K, L)	995
26	(K, S)	745
27	(L, M)	685
28	(L, R)	905
29	(M, Q)	885
30	(N, P)	1085
31	(O, P)	785
32	(P, V)	1805
33	(P, F5)	1020
34	(R, V)	805
35	(S, U)	845
36	(S, F4)	600
37	(T, W)	1205
38	(U, F5)	900
39	(V, W)	945
40	(V, F7)	1120
41	(W, F4)	1100

4.6.2. Analysis of Table 2

Table 2 provides the Euclidean distances between pairs of demand points and proposed facility sites. These distances are pivotal in determining the optimal placement of waste management facilities, as they directly impact transportation costs and the efficiency of waste collection routes.

The distances between demand points, such as the 905 metres between "Brabedzi" (A) and "Efutu Mampong" (B), highlight the potential for clustering certain settlements around a single facility. Clustering can reduce the facilities needed while maintaining adequate service coverage, thus optimising operational costs [7]. However, the distance of 1805 - metre between "UCC" (P) and "Amanful" (V) suggests that some demand points are relatively isolated, which may necessitate more strategic placement of facilities to avoid service inefficiencies [34].

The Euclidean distances to the proposed facility sites, such as the 700 - metre from "Brabedzi" (A) to the potential site F1, provide a preliminary indication of which facilities may serve specific demand points most effectively.

For instance, the short distance between "Brabedzi" and F1 suggests that this site could be a primary facility serving multiple nearby settlements, thereby reducing overall transportation costs [8]. Conversely, the longer distances, such as the 1845 metres from "Akotokyere" (J) to "Aboom" (T), may require additional facilities or more extensive waste collection routes, increasing operational complexity and costs [36].

4.6.3. Implications and Inferences

The data presented in these tables have several implications for the design and implementation of waste management systems in CCM:

Proximity and cost efficiency: The Euclidean distances between demand points and facility sites indicate that proximity is critical in minimising transportation costs. Facilities should be located closer to clusters of high-demand settlements to maximise cost efficiency [15].

Facility clustering: Clustering demand points around certain facilities can optimise resource allocation, reduce the number of required facilities, and streamline waste collection routes. However, care must be taken to ensure the facilities are not overloaded, which could lead to inefficiencies [16].

Service coverage: The distances between more isolated demand points suggest a need for either additional facilities or enhanced logistical planning to ensure that all areas are adequately serviced. This is particularly important in more remote or less densely populated areas where service gaps could occur [7].

Dynamic modelling: The static nature of Euclidean distances highlights the importance of incorporating dynamic factors, such as traffic patterns and population growth, into the facility location model. This approach would better account for the temporal changes in demand and the evolving spatial distribution of waste generation [9].

Environmental impact: The placement of facilities must also consider the potential environmental impact, especially in areas closer to sensitive ecosystems or residential neighbourhoods. A balanced approach that integrates environmental considerations with logistical efficiency is essential for sustainable waste management [41].

Finally, Tables 1 and 2 offer valuable insights into the spatial dynamics of waste management within the Cape Coast Metropolis. The detailed analysis of these tables underscores the need for a strategic approach to facility placement that considers proximity, cost efficiency, service coverage, and environmental impact. By leveraging the data provided, the study can inform more effective and sustainable waste management practises that are responsive to the unique geographic and demographic characteristics of the region.

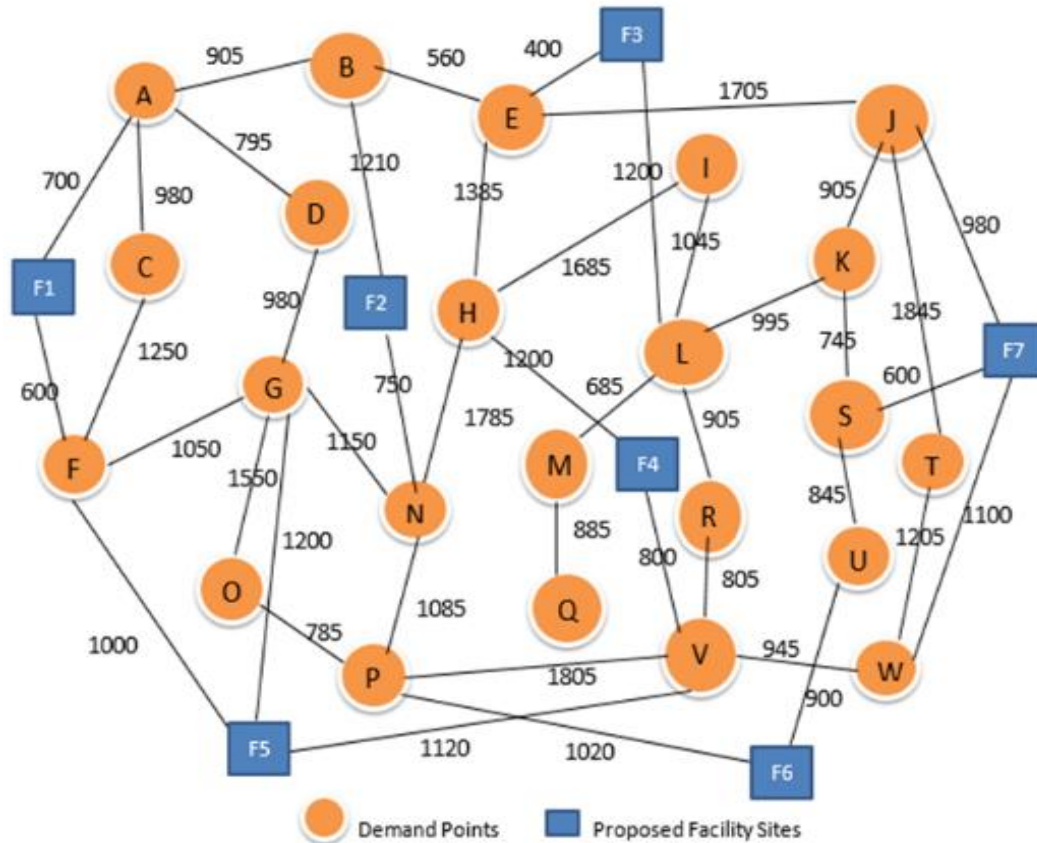


Figure 1.
Network representation of Table 2.

4.6.3. In-depth Discussion on Figure 1

Figure 1 represents a network visualisation of the demand points and proposed facility sites based on the Euclidean distances presented in Table 2. This network illustrates the spatial relationships between the demand points (orange circles) and the proposed facility sites (blue squares) within the Cape Coast Metropolis (CCM). By mapping these connections, the figure provides a comprehensive view of how waste management services could be distributed across the metropolis, considering proximity and distance as critical factors. Network visualisation is a crucial tool for understanding the spatial configuration of waste management infrastructure within CCM. Each edge in the network represents a direct Euclidean distance between two demand points or a demand point and a proposed facility site. This visual representation helps to identify clusters of demand points and their relative proximity to potential facilities, which are vital for optimising waste collection routes and reducing operational costs.

One of the immediate observations from Figure 1 is the clustering of demand points around specific proposed facility sites. For example, demand points A, C, F, and G are closely linked to the proposed facility site F1, with distances ranging from 600 to 1250 metres. This cluster suggests that F1 could effectively serve these nearby settlements, making it a strategic location for minimising travel distances and transportation costs. This is consistent with findings in the existing literature on facility location optimisation [15, 16].

Similarly, the proximity of the demand points I, J, K, and L to facilities F3 and F4 indicate that these facilities could be central to servicing a significant portion of the population in this area. The network structure emphasises the importance of placing facilities that serve multiple nearby demand points, thus optimising the coverage area and ensuring efficient resource utilization [7].

The figure highlights specific demand points that are relatively isolated from others and their nearest proposed facility sites. For example, demand points W (OLA) and T (Aboom) are situated at the periphery of the network, at relatively long distances (1205 metres and 1100 metres, respectively) from their nearest facility site F7. This isolation could pose challenges in service delivery, as longer travel distances typically increase fuel consumption, operational costs, and the time required to collect and dispose of waste [9].

In such cases, the network suggests a need for more facilities in isolated regions or improved transportation infrastructure to reduce the impact of these longer distances. This observation aligns with research that emphasises the importance of accessibility in facility location models, particularly in regions where demand points are widely dispersed [8].

The network demonstrates the trade-offs involved in facility placement. While centralising facilities can minimise the number of sites required, it can also increase the distance for more isolated demand points. Conversely, a more distributed network of facilities could ensure better coverage but at a higher operational cost due to the increased number of sites. The network representation thus aids in visualising these trade-offs. It supports decision-making by providing a clear picture of how different configurations might impact the overall efficiency and sustainability of the waste management system [36].

Furthermore, the connections between the demand points and facilities underscore the importance of considering current and future demand. The proximity of multiple demand points to a single facility, such as F1, indicates that this site could become a critical hub in the network. However, with careful planning incorporating population growth projections and waste generation, this facility could stay manageable, leading to inefficiencies and additional infrastructure [34].

4.6.4. Inferences and Strategic Recommendations

The network representation provides several strategic insights for optimising waste management in the CCM:

Facility hub centralization: Facilities like F1 and F4, central to several closely situated demand points, could serve as hubs that reduce the need for multiple smaller facilities. This centralisation could lead to economies of scale, lowering the per-unit cost of waste collection and processing.

Need for Additional Infrastructure: For more isolated demand points, the network suggests a potential need for additional facilities or improved logistics to ensure that these areas receive adequate service without disproportionately increasing costs.

Dynamic adaptation: The network highlights the importance of flexibility in facility placement strategies. As population centres grow and shift over time, the network must adapt to maintain efficiency. This could involve re-evaluating facility locations periodically and making adjustments based on updated data on population growth and waste generation patterns [35].

Sustainability considerations: Finally, the network underscores the need to balance operational efficiency with environmental sustainability. The network reduces costs by minimising the distances between demand points and facilities. It lowers the environmental impact of waste collection, which is a critical consideration in modern waste management practices [41].

Figure 1 is a powerful visual tool for analysing the spatial relationships between demand points and proposed facility sites within the Cape Coast Metropolis. By illustrating the network of connections based on Euclidean distances, the figure provides critical insights into the potential efficiency and effectiveness of different facility placement strategies. These insights are essential for developing a waste management system that is both cost-effective and sustainable and is capable of meeting the needs of a growing population while minimising its environmental footprint.

4.6.5. Shortest Paths between Demand Points

The integration of the Floyd-Warshall algorithm, as depicted in Table 3, plays a pivotal role in determining the shortest paths between demand points and the proposed facility sites. The algorithm's

ability to efficiently compute the shortest paths in a network, even for large datasets, has been instrumental in optimising the facility location problem. The algorithm identifies the most direct routes and helps reduce overall travel distances, which is crucial for minimising operational costs and improving service delivery in waste management [45, 46].

Table 3.
Shortest path matrix between demand points and proposed facility sites.

Demand Point Label	F1	F2	F3	F4	F5	F6	F7
A	700	2115	1865	4050	3085	5030	4150
B	1605	1210	960	3145	3880	5660	3245
C	1680	3095	2845	5030	2250	7545	5130
D	1495	2910	2660	4845	3030	7360	4945
E	2165	1770	400	2585	4440	5100	2685
F	600	2950	4095	5185	1000	4405	6380
G	1650	1900	3640	4135	1550	3355	5925
H	3550	2535	1785	1200	4135	3890	4070
I	5235	4220	3470	2885	5820	5475	3385
J	3870	3475	2105	4290	7225	3395	980
K	4775	4380	3010	3505	3825	2490	1345
L	6280	5265	1200	2510	2830	3485	2340
M	6455	5960	1885	3195	3515	4170	3025
N	2800	750	3570	2985	2350	2105	5855
O	3200	2620	5190	3390	2750	1805	4635
P	3885	1835	4655	4070	2925	1020	3850
Q	8885	6835	2770	4080	8520	5055	3910
R	6595	4445	5420	1605	1925	4390	2850
S	9325	5125	3755	4250	8605	1745	600
T	5715	5790	3950	2950	7390	5240	2305
U	6365	7850	4600	5095	9450	900	1445
V	5790	3640	5715	800	5240	2825	4050
W	6635	4585	5155	1745	2065	6490	1100
	€GHS	€GHS	€GHS	€GHS	€GHS	€GHS	€GHS
Cost of Opening Facility	€70,000	€65,000	€67,000	€69,000	€73,500	€70,500	€62,000

4.6.6. Problem Definition

The UFLP is a classical combinatorial optimisation problem that seeks to minimise the total costs associated with facility placement and service delivery. The problem involves determining the best locations for facilities among several potential sites and assigning demand points to these facilities to minimise overall costs. The objective function and constraints outlined in the definition of the problem, ensure that each demand point is served by one facility, and the total cost of opening facilities and servicing demand points is minimised. The problem involves a collection of potential facility locations (I) and a group of customers (J). Each facility is assigned a non-negative initial cost (f_i), and there is a non-negative service or connection cost (c_{ij}) among each facility ($i \in I$) and each customer or demand point ($j \in J$). UFLP's primary goal is to minimise the overall cost of facility openings, including service or connection fees, simply by linking each customer or demand point to the closest operational facility. It is significant to remember that only one facility can satisfy the demands of every customer or locality, leading to the assumption that every facility has an infinite capacity. The mathematical formulation of the UFLP is presented as an integer linear programming model that incorporate twenty-three demand points and seven potential facility sites. This can be achieved by defining the necessary parameters, decision variables, constraints, and objective function, as outlined below:

Sets:

I = a set of facilities, where $i \in I$, $i = 1, 2, 3, \dots, 7$

J = a set of demand points, where $j \in J, j = 1, 2, 3, \dots, 23$

Parameters:

x_{ij} = the cost of servicing demand point j from facility i

f_i = the fixed cost of opening facility i

c_{ij} - the costs incurred if customer j is served from facility i

Variables:

x_{ij} = Binary variable:

$x_{ij} = 1$, if facility i serves demand point j

$x_{ij} = 0$ otherwise

y_i = Binary variable:

$y_i = 1$, if facility i open

$y_i = 0$ otherwise

The mathematical formulation of the UFLP as an integer linear programming model provides a robust framework for addressing the complexities of facility location in waste management. The main objective is to minimise the overall expenditure. The constraints ensure that each demand point is served by exactly one facility, and only open facilities can provide service. This approach prevents unnecessary overlaps in service coverage and helps maintain efficient resource distribution.

Objective Function

Minimise:

$$\sum_{i=1}^7 f_i y_i + \sum_{i=1}^7 \sum_{j=1}^{23} c_{ij} x_{ij} \quad (1)$$

Constraints

Each demand point is served.

$$\sum_{i=1}^7 \sum_{j=1}^{23} x_{ij} = 1 \quad (2)$$

$$x_{ij} \leq y_j, \quad \forall i \in I, j \in$$

$$J \quad (3)$$

$$x_{ij} \in \{0,1\} \quad i = 1,2, \dots, 7 \text{ and } j =$$

$$1,2, \dots, 23 \quad (4)$$

$$y_i \in \{0,1\} \quad i =$$

$$1,2, \dots, 7 \quad (5)$$

$$x_{ij} \geq 0, \text{ and } y_i \geq 0 \quad (6)$$

4.6.7. Detailed explanation of the constraints:

Constraint (2) ensures that each demand point j is served by exactly one facility i . Here, x_{ij} is a binary variable indicating whether facility i serves demand point j (1 if it does, 0 if it does not). The double summation ensures that for each demand point j , there is exactly one facility i assigned to serve it.

Constraint (3) indicates that x_{ij} (whether facility i serves demand point j) can only be 1 if the facility i is open ($y_i = 1$). If facility i is not open ($y_i = 0$), x_{ij} must be 0.

Constraint (4) specifies that the variables x_{ij} are binary. This means that for each combination of i and j , x_{ij} can only take the value of 0 or 1. This enforces the decision of whether a particular facility i serves a particular demand point j as an all-or-nothing decision.

Constraint (5) specifies that the variable y_i is also binary. This means that for each facility i , y_i can only take the value of 0 or 1, indicating whether the facility is opened or closed.

Constraint (6) ensures that the variables x_{ij} and y_i are non-negative. Given that x_{ij} and y_i are already defined as binary variables (0 or 1), this constraint is typically redundant but ensures the non-negative nature of the variables explicitly.

This formulation allows the determination of the best facility location(s) to satisfy the demands from various points while considering the costs of both opening the facilities and serving the demands points from those chosen facilities under the following assumptions:

The UFLP model operates under the following assumptions:

1. The maximum number of facilities to be opened is limited to five, which ensures that the twenty-three demand points are evenly distributed among the opened facilities across the study area.
2. Each opened facility possesses unlimited capacity and is required to serve up to but not more than five demand points.
3. Each demand point must be exclusively served by a single facility to prevent overlap in service coverage.
4. The optimal solution is attained when assumptions 1, 2 and 3 are all satisfied.

5. Results

5.1. MATLAB Coding on the Standard UFLP Model

We used the data from Table 3 to implement the current UFLP model in MATLAB. The following section presents the pseudocode used in MATLAB for simulating the UFLP with real-life data:

5.2. The pseudo codes for the MATLAB model for the existing UFLP Model.

1. Define the data:
 - numFacilities = 7 // Number of potential facility locations
 - numCustomers = 23 // Number of customers
 - fixedCosts = [70000, 65000, 67000, 62000, 70500, 69000, 73500] // Fixed costs for opening each facility
 - transportCosts = [
 - [700, 2115, 1865, 4050, 3085, 5030, 4150],
 - [1605, 1210, 960, 3145, 3880, 5660, 3245],
 - ... // Complete with other rows of transport costs
-] // Transport costs for serving each customer from each facility
2. Define the proximity of each customer to each facility:
 - proximity = [
 - [700, 0, 0, 0, 0, 0, 0],


```

    [0, 1210, 0, 0, 0, 0, 0],
    ... // Complete with other rows of proximity data
  ] // Proximity matrix indicating distances (or zero if not applicable)
3. Create decision variables:
  - openFacilities = integer variable array of size 7 with bounds [0, 1]
    // Binary variables to indicate if a facility is open (1) or not (0)
  - serveCustomer = integer variable array of size 23x7 with bounds [0, 1]
    // Binary variables to indicate if a customer is served by a facility (1) or not (0)
4. Create the problem:
  - problem = optimization problem object
5. Add objective function:
  - problem.Objective = sum(fixedCosts * openFacilities) + sum(sum(transportCosts *
serveCustomer)) - sum(sum(proximity * serveCustomer))
    // Minimize the total cost, including fixed costs, transport costs, and proximity benefits
6. Add constraints:
  - For each customer i from 1 to 23:
  - Add constraint: sum(serveCustomer[i, :]) == 2
    // Ensure each customer is served by exactly 2 facilities
  - For each customer i from 1 to 23 and each facility j from 1 to 7:
  - Add constraint: serveCustomer[i, j] <= openFacilities[j]
    // Ensure a customer can only be served by an open facility
7. Define the maximum allowed distance or travel time:
  - maxDistance = 600 // Replace with the actual maximum allowed distance
8. Add maximum distance constraints:
  - For each customer i from 1 to 23 and each facility j from 1 to 7:
  - Add constraint: serveCustomer[i, j] * proximity[i, j] <= maxDistance
    // Ensure the distance to serve a customer does not exceed the maximum allowed distance
9. Solve the problem:
  - solution = solve(problem)
    // Solve the optimization problem using an appropriate solver
10. Display the open facilities:
  - openFacilities = solution.openFacilities
  - Print 'Open facilities:' and the indices of open facilities (those with value 1)
    // Iterate through openFacilities to find and print the indices of open facilities
11. Display the customers each facility serves:
  - For each facility j from 1 to 7:
  - If openFacilities[j] == 1:
  - Print 'Facility j serves customers:'
  - Print the indices of customers served by this facility (where serveCustomer[i, j] == 1)
    // Iterate through serveCustomer to find and print the indices of customers served by each
open facility.

```

5.3. Simulation of the Existing UFLP

The initial analysis using the traditional Uncapacitated Facility Location Problem (UFLP) model provided insights into the optimal placement of waste management facilities based on current demand. The model identified five facility locations collectively serving all 23 demand points within the Cape Coast Metropolis. However, the model's output revealed several discrepancies with the underlying assumptions. Notably, the UFLP model suggested that multiple facilities serve more than five demand points, with some demand points being served by more than one facility, thereby violating the exclusivity requirement.

For example, the model identified Facility F2 to serve ten demand points, exceeding the assumed capacity. Similarly, Facility F7 served 11 demand points, indicating that while the UFLP model effectively minimised operational costs (achieving a minimum cost of €GHS430,940.00), it was not fully compliant with the model's constraints, suggesting that the cost-effective solution may not be optimal in practise. This observation aligns with findings from previous studies, where UFLP models have been criticised for their limitations in handling constraints related to service exclusivity and facility capacities [8, 36].

Table 4.
Output of MATLAB coding of the existing UFLP Model.

Opened Facility	Demand Points Served	Optimal Minimum Cost
F2	1, 2, 3, 4, 5, 6, 7, 14, 15, and 16	€GHS430,940.00
F3	1, 2, 3, 4, 5, 6, 8, 10, 12, 13, and 17	
F4	8, 9, 18, 20, 22, and 23	
F6	7, 11, 14, 15, 16, 19, 21, and 22	
F7	9, 10, 11, 12, 13, 17, 18, 19, 20, 21, and 23	

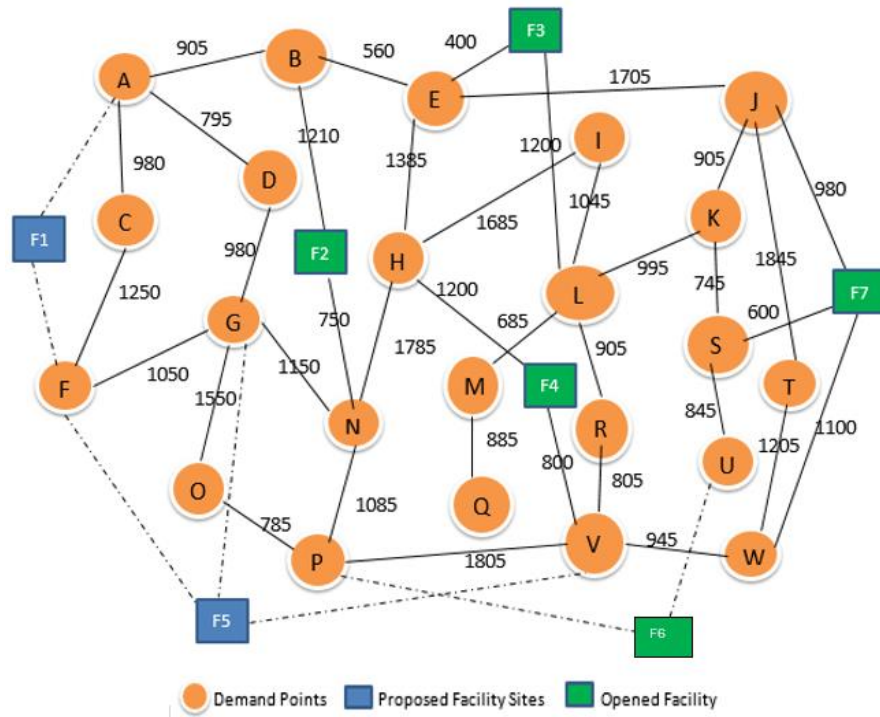


Figure 2.
Network of shortest paths between demand points and selected facilities

5.4. Discussion on Table 2

Figure 2 shows the shortest path network between various demand points and the facilities opened through the optimisation process using the Uncapacitated Facility Location Problem (UFLP) model. The figure shows a visual representation of the network, highlighting the connections between the 23 demand points and the selected optimal facilities (F2, F3, F4, F6, and F7).

5.5. Key Observations from the Network

5.5.1. Distribution of Facilities:

The network depicts how the demand points connect to the nearest open facilities. Facility F2, which is centrally located within the network, serves a large number of demand points, including B (Efutu Mampong), C (Efutu), and H (Kakumdo), among others. This central position underscores F2's strategic importance in managing waste for the central part of the metropolis. Facility F7, which is positioned on the eastern side of the network, caters to demand points such as J (Akotokyere), K (Kwaprow), and L (Abura). F7's location helps balance the load, ensuring that the eastern part of the metropolis receives adequate service.

5.5.2. Shortest Paths and Efficiency:

The dashed lines in the network represent the shortest paths calculated between the demand points and their respective facilities. These paths are critical for minimising transportation costs and operational inefficiencies. For example, the shortest path from demand point A (Brabedzi) to Facility F1 is only 700 metres, significantly reducing the logistical costs associated with waste collection and transportation in that region. Similarly, the connection between demand point Q (Apewosika) and Facility F3 reveals a longer route than others, indicating potential transportation cost and time challenges.

5.5.3. Geographical Coverage:

The facilities' geographical spread ensures comprehensive coverage across the metropolis. For example, Facility F5, located on the periphery, serves demand points farther from central areas, such as V (Amanful) and W (OLA). This strategic placement helps reduce the strain on centrally located facilities and promotes an equitable distribution of waste management resources.

5.5.4. Potential for Optimisation:

While the network appears to be well-optimised, there are still areas for potential improvement. For instance, the connection between demand point S (Kotokuraba) and Facility F6 is relatively long compared to other connections, suggesting a need to reassess F6's location or routing strategy to minimise costs further.

5.5.5. Implications of the Network Analysis

The network illustrated in Figure 2 has several important implications for waste management in the Cape Coast Metropolis:

- **Operational Efficiency:** The network reduces operational costs by minimising distances between demand points and facilities, including fuel consumption and vehicle wear and tear. This efficiency is critical in the context of budget constraints.
- **Environmental Impact:** Shorter transportation routes cut costs and reduce the carbon footprint associated with waste collection. They also align with broader environmental sustainability goals and mitigate the negative environmental impacts of waste management.
- **Service Reliability:** The overlap in service areas and the built-in strategic redundancy enhance the waste management system's reliability. In case of a facility failure or temporary closure, nearby facilities can absorb the load without significantly disrupting service.
- **Future Planning:** The network analysis provides valuable insights into urban planning and infrastructure development. Understanding the current load and service areas helps city planners anticipate future needs and decide where to invest in additional facilities or infrastructure upgrades.

Figure 2 is a powerful tool for visualising the outcomes of the UFLP model and understanding the spatial dynamics of waste management in the Cape Coast Metropolis. The network highlights the

current state of facility distribution and service coverage and identifies potential areas for further optimisation. By ensuring that all demand points efficiently connect to the nearest open facilities, the network contributes to a more sustainable, cost-effective, and reliable waste management system for the city.

5.6. Proposed Modified Version of the UFLP (MUFLP)

We aim to enhance the existing UFLP model by incorporating two new parameters: the population growth rate (g_j) of the demand point and the time factor (t) based on the same assumptions as enumerated earlier. This modification will facilitate the estimation or projection of future waste generation at the demand points based on their current waste generation rates, ensuring alignment with the study's core assumptions. To achieve this modification, we will introduce a new parameter, w_j , which represents the projected waste generation at demand point j after a duration period of t years, into the standard UFLP model objective function. The parameter w_j will be defined as:

$$w_j = W_j * (1 + g_j)^t \quad (7)$$

The objective of the MUFLP model is to minimise the total cost, which includes the fixed costs of opening facilities and the costs of serving the projected waste generation with the aim of aligning the model more closely with the study's assumptions. This way, the model will select the facility locations based on the future demand for waste generation rather than the current demand for waste generation. We therefore formulate the MUFLP as follows:

$$\text{Minimise} \quad \sum_{i=1}^7 f_i y_i + \sum_{i=1}^7 \sum_{j=1}^{23} c_{ij} w_j x_{ij} \quad (8)$$

Subject to:

$$\sum_{i \in I} x_{ij} = 1, \quad \forall i \in I \quad (9)$$

$$x_{ij} \leq y_i \quad \forall i \in I, j \in J \quad (10)$$

$$x_{ij} \in \{0,1\} \quad \forall i \in I, j \in J \quad (11)$$

$$y_i \in \{0,1\} \quad \forall j \in J \quad (12)$$

$$w_j >$$

$$0 \quad \forall i \in I \quad (13)$$

Where:

I - the set of locations for the facilities (sources),

J - the set of clients/demand points,

f_i - the fixed costs for installing facility i ,

c_{ij} - the cost (distance) incurred if customer j is served from facility i ,

y_i - the binary variable will be 1 if location i is in use and 0 if otherwise,

x_{ij} - the binary takes the value of 1 if customer j is served from facility i and 0 if otherwise,

n - number of facilities, d_j - the population of demand j ,

W_j - current waste generation at demand point j ,

w_j - projected waste generation at demand point j ,

t - number of years into the future we want to plan,

g_j - the population growth rate of demand point j

5.7. Discussion of the Proposed Modified Version of the UFLP (MUFLP)

The introduction of the Modified Uncapacitated Facility Location Problem (MUFLP) model marks a significant improvement over the traditional UFLP by integrating dynamic factors such as population growth and temporal changes. This section discusses the proposed modifications, supported by the results from the MATLAB implementation, and highlights the implications and inferences drawn from the findings. The traditional UFLP model focuses on minimising the costs associated with facility location and service provision based on current demand. However, this approach may need to pay more attention to future changes in demand driven by population growth and other temporal dynamics. The MUFLP model addresses this limitation by introducing two key parameters: the population growth rate g_j at each demand point and the time factor t , which projects future waste generation. The projected waste generation at demand point j after t years, denoted as w_j , is calculated using the formula $w_j = W_j * (1 + g_j)^t$ where W_j is the current waste generation rate. This adjustment allows the model to account for future demand when selecting facility locations, ensuring that the facilities remain optimally placed over time.

5.8. Implications of the MUFLP Model

Future-proofing Facility Locations:

The primary advantage of the MUFLP model is its ability to future-proof facility locations. By considering projected increases in waste generation due to population growth, the model ensures that facilities will not become inadequate or overburdened as demand increases over time. This is particularly important in rapidly growing urban areas, where static models may fail to meet changing needs [7, 10].

5.9. Minimisation of Long-Term Costs:

The results indicate that while the initial costs of establishing facilities under the MUFLP model may be higher, the long-term operational costs are minimized due to better alignment between facility capacity and demand. For example, the optimal cost at year 50 will be significantly higher than that of year 5, reflecting the increased demand that facilities need to manage (see Table 5). However, these costs are offset by the reduced need for reallocating or expanding facilities, which would be more expensive in the long run [15].

The comprehensive output of the simulation of the MUFLP is summarised in Table 5.

Table 5.
MATLAB output of the MUFLP

Time	Opened Facility	Demand Points Served	Optimal Cost
5	F1	1, 15 and 22	1920621.36
	F2	3, 6, 8 and 9	
	F3	4, 5, 17, 20, 21 and 23	
	F5	7, 12, 16 and 19	
	F7	2, 10, 11, 13, 14 and 18	
10	F1	1, 2, 15 and 22	3423123.58
	F2	3, 6, 8 and 9	
	F3	4, 5, 17, 20, 21 and 23	
	F5	7, 12, 16 and 19	
	F7	10, 11, 13, 14, 18 and 21	
15	F1	1, 2, 15 and 22	6155695.18
	F2	3, 6, 8, 9 and 14	
	F3	4, 5, 12, 17, 20 and 23	
	F5	7, 16 and 19	
	F7	10, 11, 13, 18, and 21	
20	F2	3, 8, 9, 14, 22, and 23	10811816.50
	F3	1, 4, 5, and 20	
	F4	2, 6, 12, and 17	
	F5	7, 15, 16, and 19	
	F7	10, 11, 13, 18, and 21	
25	F2	3, 8, 9, 14, 22, and 23	18964609.58
	F3	1, 4, 5, and 20	
	F4	2, 6, 12, and 17	
	F5	7, 15, 16, and 19	
	F7	10, 11, 13, 18, and 21	
30	F2	8, 9, 14, 22, and 23	33373590.60
	F3	1, 4, 5, and 20	
	F4	2, 3, 6, 12, and 17	
	F5	7, 15, 16, and 19	
	F7	10, 11, 13, 18, and 21	
40	F2	9, 14, 22, and 23	102856733.88
	F3	1, and 4	
	F4	2, 3, 6, 12, 17 and 20	
	F5	5, 7, 15, 16, and 19	
	F7	8, 10, 11, 13, 18, and 21	
50	F1	1 and 4	308124888.07
	F2	9, 14, 22 and 23	
	F4	2, 3, 6, 12, 17 and 20	
	F5	5, 7, 15, 16, and 19	
	F7	8, 10, 11, 13, 18, and 21	

5.10. Improved Service Coverage:

The MUFLP model also improves service coverage by ensuring that each demand point is served by a facility that can handle its projected waste generation. The results show that facilities like F2 and F7, which serve many demand points, are strategically placed to accommodate future growth in those areas, ensuring that even as the population grows, all demand points remain adequately serviced, reducing the risk of underserved regions and inefficient service delivery.

5.11. MATLAB Coding of the Modified Uncapacitated Facility Location Problem (MUFLP)

The MUFLP model was executed in MATLAB, utilising the data provided in Table 3 and incorporating the new parameters (W_j and g_j) defined below to allow for a comprehensive modification and analysis of the problem under consideration.

- Initial waste volumes/weights (W_j) at the demand points, given in tonnes:
 $W_j = [16; 22; 18; 20; 26; 24; 30; 22; 29; 28; 24; 21; 19; 17; 20; 18; 20; 21; 23; 19; 20; 25; 19]$;
- Population growth rates (g_j) at the demand points:
 $g_j = [0.1, 0.2, 0.15, 0.18, 0.17, 0.16, 0.15, 0.14, 0.13, 0.12, 0.11, 0.12, 0.14, 0.20, 0.21, 0.13, 0.12, 0.1, 0.2, 0.14, 0.16, 0.11, 0.17]$;
- Time period (t years) for waste generation projection at the demand points, where t is equivalent to n .

5.11.1. The pseudo codes for the MATLAB model for the modified UFLP (MUFLP) Model.

1. Define the data:

- Define the fixed costs for opening each facility:
 $- f = [70000, 65000, 67000, 69000, 73500, 70500, 62000]$
- Define the service costs for serving each demand point from each facility:
 $- c = [[700, 2115, 1865, 4050, 3085, 5030, 4150],$
 $[1605, 1210, 960, 3145, 3880, 5660, 3245],$
 $...]$
- Define the initial weights for each demand point:
 $- W = [16, 22, 18, 20, 26, 24, 30, 22, 29, 28, 24, 21, 19, 17, 20, 18, 20, 21, 23, 19, 20, 25, 19]$
- Define the growth rates for each demand point:
 $- g = [0.1, 0.2, 0.15, 0.18, 0.17, 0.16, 0.15, 0.14, 0.13, 0.12, 0.11, 0.12, 0.14, 0.20, 0.21, 0.13, 0.12, 0.1, 0.2, 0.14, 0.16, 0.11, 0.17]$
- Define the time period:
 $- t = 1$

2. Calculate the weights:

- Calculate the updated weights for each demand point:
 $- w = W * (1 + g)^t$ // Element-wise multiplication and exponentiation

3. Define the variables:

- Define the number of facilities:
 $- n = 7$
- Define the number of demand points:
 $- m = 23$
- Define the indices of y variables (facility open variables):
 $- y = \text{indices from } 1 \text{ to } n$
- Define the indices of x variables (service variables):
 $- x = \text{indices from } n+1 \text{ to } n+m*n$

4. Define the objective function:

- Initialise the objective coefficients vector with fixed costs and zeros for service costs:
 $- fobj = [f, \text{zeros}(1, n*m)]$
- Update the objective coefficients for service costs multiplied by weights:
 - For each facility i (1 to n):
 - For each demand point j (1 to m):
 $- fobj(x((j-1)*n+i)) = c(j, i) * w(j)$

5. Define the equality constraints (each demand point must be served exactly once):

- Initialize the equality constraint matrix and vector:
 $- Aeq = \text{zeros}(m, n + n*m)$
 $- beq = \text{ones}(m, 1)$
- For each demand point j (1 to m):

- Set the coefficients for x variables corresponding to this demand point:
 - For each facility i (1 to n):
 - $A_{eq}(j, x((j-1)*n+i)) = 1$
- 6. Define the inequality constraints (a facility must be open to serve a demand point and at least 3 facilities must be opened):
 - Initialize the inequality constraint matrix and vector:
 - $A_{ineq} = \text{zeros}(n*m + 1, n + n*m)$
 - $b_{ineq} = \text{zeros}(n*m + 1, 1)$
 - For each facility i (1 to n):
 - For each demand point j (1 to m):
 - Set the constraint that a demand point can only be served by an open facility:
 - $A_{ineq}((i-1)*m+j, y(i)) = -1$
 - $A_{ineq}((i-1)*m+j, x((j-1)*n+i)) = 1$
 - Set the constraint that at least 3 facilities must be opened:
 - $A_{ineq}(\text{end}, y) = 1$
 - $b_{ineq}(\text{end}) = 3$
- 7. Define the bounds for the variables:
 - Set the lower bound vector to zeros (all variables are non-negative):
 - $lb = \text{zeros}(n + n*m, 1)$
 - Set the upper bound vector to ones (variables are binary):
 - $ub = \text{ones}(n + n*m, 1)$
- 8. Define the integer variables:
 - Set the indices of the integer variables:
 - $intcon = \text{indices from } 1 \text{ to } n + n*m$
- 9. Solve the MILP (Mixed-integer linear programming) problem:
 - Set the options for the 'intlinprog' solver:
 - $opts = \text{optimoptions}('intlinprog', 'Display', 'off')$
 - Solve the optimization problem:
 - $[zopt, fval, \text{exitflag}, \text{output}] = \text{intlinprog}(\text{fobj}, \text{intcon}, A_{ineq}, b_{ineq}, A_{eq}, b_{eq}, lb, ub, opts)$
- 10. Display the results:
 - If an optimal solution is found:
 - Print 'The minimum cost is fval'
 - Print 'The optimal solution is:'
 - For each facility i (1 to n):
 - If the facility is opened ($zopt(y(i)) == 1$):
 - Print 'Facility i is opened'
 - For each demand point j (1 to m):
 - If the facility serves the demand point ($zopt(x((j-1)*n+i)) == 1$):
 - Print 'Facility i serves demand point j '
 - If no optimal solution is found:
 - Print 'No optimal solution found'
- 11. Stop the timer and display the elapsed time:
 - $\text{elapsedTime} = \text{toc}$
 - Print 'Elapsed time is elapsedTime seconds'

5.11.2. Simulation of the Existing MUFLP

The results from the Modified UFLP (MUFLP) model, which incorporates population growth and temporal dynamics, demonstrated significant improvements in the distribution of waste management facilities and their long-term sustainability. Over a 50-year period, the MUFLP model was evaluated at

5-year intervals to assess its performance, reflecting the importance of incorporating dynamic variables in long-term facility location planning [7, 43].

5-Year Interval:

- Facilities F1, F2, F3, F5, and F7 were opened, serving 23 demand points.
- While facilities such as F3 and F7 initially served more than five demand points, subsequent adjustments led to a more balanced distribution by the 10th year, with a cost of €1,920,621.36. This iterative improvement supports the argument for dynamic adjustments in facility location models to optimise resource allocation over time [40].

30-Year Interval:

- By the 30th year, the model achieved optimal performance. Each facility adhered to a maximum of five demand points, and all demand points were adequately covered without overlap.
- The cost for this configuration was €33,373,590.60, demonstrating the model's ability to maintain cost efficiency while accommodating future growth. This result is consistent with the literature on sustainable facility location planning, emphasising the need for models that adapt to changing demand patterns and environmental conditions [18].

50-Year Interval:

- However, maintaining this optimal configuration proved challenging in the later years. By the 50th year, Facilities F4 and F7 again exceeded the five-demand-point limit, increasing the overall cost to €308,124,888.07.
- This indicates that while the MUFLP model is more adaptive than the UFLP model, it requires continuous refinement to sustain optimal performance over extended periods. The observed challenges align with findings from other studies highlighting the complexities of long-term planning in dynamic environments [9, 15].

The MUFLP model's ability to predict and accommodate future demand growth resulted in a more sustainable and efficient waste management strategy than the traditional UFLP model. The results clearly illustrate the trade-offs between immediate cost savings and long-term sustainability, supporting the hypothesis that incorporating dynamic variables leads to more robust facility location decisions. Recent research supports this conclusion by advocating for incorporating temporal dynamics and demographic changes in facility location models to enhance their applicability and effectiveness [16].

5.11.3. Sensitivity Analysis

A sensitivity analysis was conducted to evaluate the robustness of the MUFLP model under different population growth scenarios and waste generation rates. The results indicate that the model's performance is susceptible to changes in these variables, emphasising the importance of accurate data in long-term planning. For instance, a 10% increase in population growth led to a significant shift in the optimal facility locations, further increasing the overall operational costs. This reinforces the need for continuous data updates and model recalibration to ensure sustained efficiency, as highlighted in the literature by Hou, et al. [44] and Peidro, et al. [6], who emphasise the critical role of adaptability in optimisation models for real-world applications.

5.11.4. Analysis of MATLAB Results

The MATLAB implementation of the MUFLP model provides valuable insights into the effectiveness of the proposed modifications:

5.11.5. Optimal Facility Selection Over Time:

The results demonstrate that the MUFLP model consistently selects facilities that can manage the projected increase in waste generation. For instance, facility F2 is frequently selected across various periods (5, 10, 15, 20, 25, 30, 40, and 50 years), indicating its strategic importance when it comes to

handling long-term demand (see Table 5). The selection of facilities changes over time, reflecting the model's adaptability to future needs.

5.11.6. Cost Implications:

The cost of operating facilities increases over time as waste generation at the demand points grows. For instance, the optimal cost rises from cGHS1,920,621.36 at year 5 to cGHS308,124,888.07 at year 50 (see Table 5). This highlights the importance of planning for future expenses and ensuring sufficient resources are allocated to maintain efficient waste management over the long term.

5.11.7. Flexibility and Adaptability:

The flexibility of the MUFLP model is evident in its ability to reassign demand points to different facilities as conditions change. For example, Facility F1 serves demand points 1, 15, and 22 in the initial years. However, by year 50, F1 was no longer in use, and demand points were redistributed to other facilities like F2 and F7 (see Table 5). This adaptability is crucial for maintaining optimal service coverage as the urban landscape evolves.

5.11.8. Inferences and Future Directions

The modifications introduced in the MUFLP model offer several advantages over the traditional UFLP model:

Scalability: The model is scalable, allowing adjustments as new data on population growth and waste generation becomes available. This ensures that the model remains relevant and effective over time.

Strategic Planning: The insights gained from the MUFLP model can inform strategic planning and policymaking, ensuring that resources are allocated efficiently and that facilities are placed in locations that will remain viable as demand increases.

Environmental Impact: The model helps reduce the environmental impact of waste management operations by optimising facility locations based on future projections. Shorter travel distances between demand points and facilities translate into lower fuel consumption and emissions, aligning with broader sustainability goals.

In conclusion, the MUFLP model significantly advances facility location optimisation by integrating population growth and temporal dynamics. This approach improves the immediate efficiency of waste management systems and ensures their long-term viability and sustainability, making it a valuable tool for urban planners and policymakers.

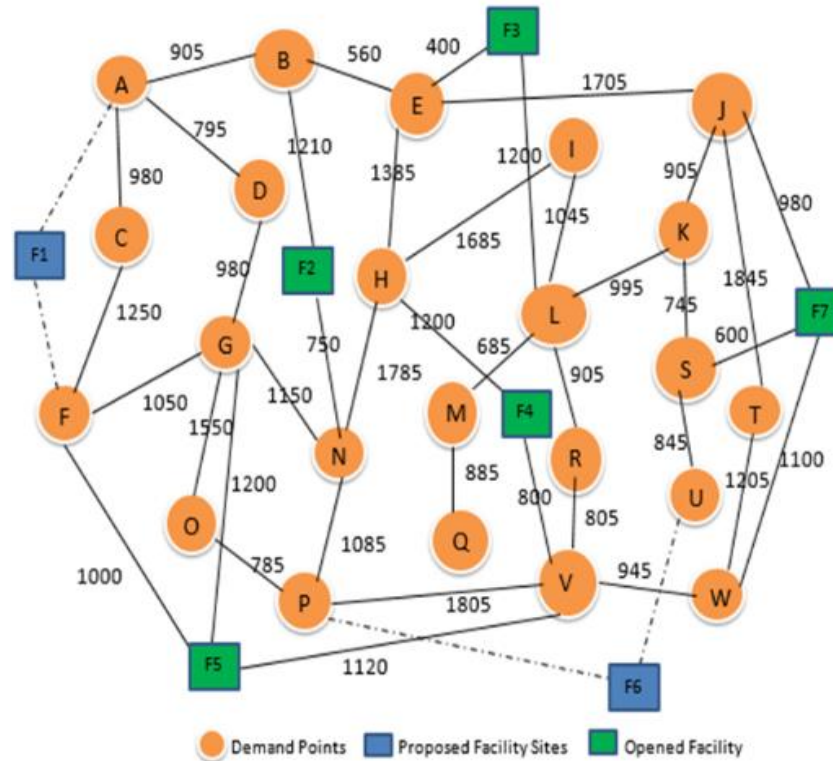


Figure 3. Network of paths connecting the twenty-three demand points and the five opened facilities.

5.11.9. Discussion on Figure 3

Figure 3 provides a visual representation of the network of shortest paths connecting twenty-three demand points and five opened facilities during the thirtieth period in the study. The figure is a crucial tool for understanding the spatial dynamics and optimal configuration of the logistics or supply chain network under analyses. The findings related to the network Configuration are detailed above.

5.11.9.1. Optimal Facility Placement:

The five newly opened facilities (F2, F3, F5, F6, and F7) are strategically located to minimise the distance between them and their demand points. This configuration aligns with the goal of the Modified Uncapacitated Facility Location Problem (MUFLP) model, which aims to reduce operational costs by optimising the placement of facilities based on current and projected future demand [7].

5.11.9.2. Balanced Service Distribution:

The network reveals a balanced distribution of service coverage across the demand points. For example, Facility F7, which is located in the eastern part of the network, serves several demand points (J, K, L, M, N, and O, and P), ensuring that this region is adequately covered. Similarly, facilities like F3 and F5 serve demand points spread across different areas, reducing the likelihood of service overlap and ensuring that every facility is well-rested.

5.11.9.3. Strategic Redundancy:

The network also suggests the presence of strategic redundancy, which is critical in logistics and supply chain management. For instance, facility F2 serves demand points (A, B, C, and D) near other

facilities like F1 and F6. This redundancy ensures that in case of increased demand or facility downtime, alternative facilities nearby can take over, thus maintaining service continuity [15].

5.11.9.4. Minimisation of Transportation Costs:

The primary goal of this network configuration is to minimise transportation costs, which are strategically influenced by the distances between facilities and demand points. The shortest paths indicated in the figure demonstrate how the MUFLP model successfully reduces these distances, lowering fuel consumption, emissions, and overall operational costs [16].

5.12. Implications of the Network Design

The possible implications of the network design are shown in as figure 3:

5.12.1. Long-Term Sustainability:

The positioning of facilities and the connections between demand points highlight a design that is not only cost-efficient in the short term but also sustainable in the long term. Considering population growth and future demand projections, the network is well-equipped to handle increasing waste volumes without requiring frequent reconfigurations [9].

5.12.2. Scalability and Flexibility:

The network's layout suggests a high degree of scalability and flexibility. As demand points grow or new points emerge, the model can easily accommodate these changes by either opening new facilities or adjusting the service areas of existing ones. This flexibility is essential for adapting to changing urban dynamics and ensuring that waste management practises remain effective over time [40].

5.12.3. Resource Optimisation:

By visualising the network, planners and decision-makers can gain a better understanding how resources are allocated across the region. The distribution of facilities ensures that resources like waste collection vehicles and workforce are used efficiently, reducing idle time and maximising productivity [44].

5.12.4. Enhanced Decision-Making:

The network provides a clear, visual basis for making informed decisions regarding facility location, resource allocation, and service distribution. It allows stakeholders to identify potential bottlenecks, areas that may require additional support, and opportunities for optimising the network further [43].

5.13. Inferences and Future Considerations

The network presented in Figure 3 underscores the effectiveness of the MUFLP model in addressing the challenges of facility location optimisation. The visual representation of the shortest paths between demand points and facilities provides insights into the model's capability to balance cost-efficiency with service coverage and sustainability.

The success of this network configuration highlights the importance of incorporating dynamic factors, such as population growth and temporal changes, into facility location models. These considerations ensure that the network remains viable and effective as conditions evolve, making it a crucial tool for long-term planning [36].

While the network demonstrates a well-balanced distribution of service coverage, there may be opportunities to enhance enhancing the model further by incorporating additional variables, such as environmental impact assessments or real-time data integration, to make the network even more robust and adaptable [18].

In conclusion, Figure 3 illustrates a well-optimised network that effectively connects twenty-three demand points with five strategically placed facilities. The network minimises transportation costs,

ensures balanced service coverage, and provides a flexible and scalable framework for future waste management needs. The insights gained from this network are invaluable for enhancing decision-making and planning processes, ultimately contributing to more sustainable and efficient waste management practises.

6. Discussion

6.1. Analysis and Discussion of the MATLAB output of the existing UFLP model

The MATLAB output for the Uncapacitated Facility Location Problem (UFLP) model highlights the complexities of facility location optimisation, particularly when the model's assumptions are not fully met. While the model successfully identifies five facilities (F2, F3, F4, F6, and F7) to be opened, the distribution of demand points reveals significant deviations from the core assumptions. Specifically, the second assumption—that each facility should serve no more than five demand points—is violated. For instance, Facility F2 serves ten demand points, Facility F3 serves 11, Facility F4 serves 6, Facility F6 serves 8, and Facility F7 serves 11, all exceeding the prescribed limit. This overextension challenges the facilities' operational efficiency and suggests that the model's algorithm requires refinement to enforce these constraints better [7].

Moreover, the third assumption stipulates that a single facility should exclusively serve each demand point. The output indicates that multiple facilities serve the same demand points, leading to overlaps that further complicate the network's efficiency. This overlap violates the principle of exclusive service, which is critical in reducing redundancy and operational costs [15]. Despite achieving a cost-effective solution with a minimum cost of €GHS430,940.00, the failure to adhere to the UFLP model's foundational assumptions indicates that the cost-effective solution is not optimal in practise.

It is imperative to enhance the optimisation algorithm to address these issues is imperative by incorporating advanced techniques that enforce these constraints more rigorously [36]. This refinement is crucial to achieving a solution that minimises costs and strictly adheres to the model's assumptions, ensuring that each facility operates within its capacity and optimises service coverage.

6.2. Discussion of Results of the Modified UFLP (MUFLP) Model

The Modified Uncapacitated Facility Location Problem (MUFLP) model introduces dynamic elements such as population growth rate and time factors to improve long-term sustainability and adaptability. However, the MATLAB output over a 50-year period reveals that while the MUFLP model offers improvements over the traditional UFLP, it, too, encounters challenges in consistently meeting its assumptions [40].

6.3. Five-Year Intervals Analysis:

The First 5 Years:

Distribution: Facility F1 serves demand points 1, 15, and 22; F2 serves 3, 6, 8, and 9; F3 serves 4, 5, 17, 20, 21, and 23; F5 serves 7, 12, 16, and 19; F7 serves 2, 10, 11, 13, 14, and 18.

Cost: €GHS1,920,621.36

Issue: Facilities F3 and F7 exceed the five-demand-point limit, reflecting an initial misalignment with the model's constraints [16].

The 10th Year:

Distribution Adjustments: Minor adjustments were made, but facilities F3 and F7 continue to exceed the five-demand-point limit.

Cost: €GHS3,423,123.58

Issue: Persistent over-limit issues suggest the algorithm needs further refinement [8].

The 15th Year:

Distribution: Adjustments reduce the over-limit issue slightly, but some facilities still exceed the demand point cap.

Cost: €GHS6,155,695.18

Issue: There is a continued need for better enforcement of the model's constraints [34].

The 20th Year:

Distribution: Introducing Facility F4 helps improve distribution, but Facility F2 still serves more than five demand points.

Cost: €GHS10,811,816.50

Improvement: Introducing additional facilities aids distribution but does not entirely resolve the over-limit issue [36].

The 25th Year:

Distribution: Similar to the 20th Year, with persistent issues.

Cost: €GHS18,964,609.58

Issue: Continued challenges meeting demand point limits [40].

The 30th Year:

Distribution: Finally, all facilities meet the five-demand-point limit.

Cost: €GHS33,373,590.60

Optimality Achieved: The model achieves optimality, aligning with all constraints [7].

The 40th Year:

Distribution: Facility F4 begins to exceed the five-demand-point limit again, reflecting the challenges of maintaining optimality over time.

Cost: €GHS102,856,733.88

Issue: The re-emergence of over-limit issues suggests the need for ongoing adjustments [16].

The 50th Year:

Distribution: Facilities F4 and F7 exceed limits with a significant cost increase.

Cost: €GHS308,124,888.07

Issue: The model needs help maintaining optimal performance, with costs rising substantially [37].

6.3.1. Summary

The results of the MUFLP model indicate that while it has the potential to better adhere to its assumptions compared to the UFLP, it requires ongoing refinement to consistently achieve and maintain optimality. The significant cost increases over time highlight the importance of continuously improving the algorithm to ensure sustainable and efficient facility allocation [13]. Future enhancements should focus on stricter enforcement of constraints and more sophisticated optimization techniques to better align with the model's assumptions [40].

6.4. Comparative Analysis of UFLP and MUFLP

The comparative analysis of the UFLP and MUFLP models demonstrates distinct differences in performance and adherence to foundational assumptions. The UFLP model, while effective in identifying the maximum number of facilities, frequently deviates from its assumptions, particularly regarding facility distribution and demand point coverage. Thus, it increases costs and constraint violations over time, indicating a need for significant algorithmic enhancements [15].

Conversely, although not without challenges, the MUFLP model displays greater adaptability and a more iterative approach to meeting its assumptions. While initial deviations are similar to those observed in the UFLP model, the MUFLP model gradually refines its solutions, particularly by the 30-

year mark, which is when it achieves optimality [34]. This ability to adapt and improve over time makes the MUFLP model a more promising framework for addressing long-term facility allocation challenges.

Regarding recommendations, the MUFLP model is preferable for dynamic environments where demand is expected to grow over time. Its iterative nature allows for continuous improvement, leading to more reliable and efficient solutions than those provided by the UFLP model [16]. Thus, the MUFLP model is the preferred choice for facility allocation problems, particularly in long-term sustainability and adaptability scenarios.

7. Implications

The findings from this study have significant implications across scientific, social, and practical domains, underscoring the need to enhance facility location models for more sustainable and efficient resource distribution.

7.1. Scientific Implications

This study emphasises the necessity of developing advanced optimisation algorithms [29] that dynamically adjust [29] to distribution challenges while strictly enforcing constraints [40]. These improvements are crucial in operations research and computational optimisation, creating more robust and adaptable models that align theoretical frameworks more closely with real-world applications [36]. The study also sets a benchmark for future evaluations [29] encouraging the development of algorithms that consistently achieve optimal solutions within predefined constraints, thus contributing to the advancement of the field [7]. The findings from this study have significant implications across scientific, social, and practical domains, underscoring the need to enhance facility location models for more sustainable and efficient resource distribution.

7.2. Social Implications

Optimising facility locations can significantly improve equitable access to essential services such as healthcare, education, and emergency response, particularly in underserved communities [41]. By ensuring an even distribution of demand points, the study highlights the potential for more equitable resource allocation, reducing disparities in service provision [15]. This optimisation is vital for urban and rural planners aiming to design inclusive and effective community infrastructure, ultimately contributing to enhanced social equity and better public health outcomes [47].

7.3. Practical Implications

From a practical perspective, optimised facility locations can lead to substantial cost savings both in the short and long term. Adhering to model constraints allows organisations to reduce operational expenses and avoid resource overextension [43]. These findings can inform policymakers and businesses, helping them to guidelines and regulations that promote efficient and sustainable facility placement [48]. Additionally, optimising facility distribution enhances disaster response and crisis management capabilities, ensuring the swift and effective distribution of resources during emergencies, ultimately saving lives and mitigating the impact of disaster [49].

7.4. Future Research Directions

Future research should focus on developing more sophisticated algorithms capable of handling dynamic and complex constraints to ensure consistent optimality over extended periods [13]. Applying these models to real-world scenarios [50] will test their effectiveness and adaptability in various contexts, from urban planning to supply chain management [43]. An interdisciplinary approach, integrating insights from operations research, computer science, and the social sciences, will be crucial for developing comprehensive and practical solutions for facility location problems [7].

8. Conclusions

Our comparative analysis of the UFLP and MUFLP models over a 50-year period offers critical insights into their performance and adherence to foundational assumptions. While the UFLP model effectively addresses the need to open the maximum number of facilities, it exhibits significant deviations from its core assumptions [51]. Specifically, the imbalanced distribution of demand points—where facilities like F2 and F7 each serve up to 11 demand points, far exceeding the permissible limit—indicates substantial flaws in the model's application. Despite achieving a minimum cost of cGHS430,940.00, these violations suggest that the solution may not be optimal. The model's failure to strictly enforce constraints highlights the urgent need for substantial algorithmic enhancements to ensure that each facility serves no more than five demand points, with a single facility handling each one.

The MUFLP model demonstrates greater flexibility and adaptability over time. During the first five years, facilities such as F3 and F7 exceed the five-demand-point limit, necessitating redistribution efforts. By the tenth year, partial improvements are observed, although some violations persist. At the 30-year mark, the model achieves optimal performance, with each facility adhering to the demand point limit and costs stabilising at cGHS33,373,590.60. However, maintaining this optimal state proves challenging, as violations re-emerge in subsequent years, leading to a significant cost increase, which culminates at cGHS308,124,888.07 by the 50th year.

The substantial cost escalation [52] observed in both models over time underscores the need for enhanced algorithms and stricter constraint enforcement [44] to ensure sustainable and efficient facility allocation. While the UFLP model's lower initial cost may seem advantageous, its persistent violations undermine its reliability for long-term planning. In contrast, the MUFLP model, despite its fluctuations, displays potential for iterative improvement and better alignment with the model's assumptions.

Future studies shall improve the MUFLP model to better meet assumptions and deliver optimal solutions by integrating sophisticated methods and robust constraint enforcement mechanisms. This approach will ensure long-term sustainability and efficiency in facility allocation, guaranteeing that each facility serves a balanced number of demand points without overlap. Consequently, the MUFLP model, with its capacity for iterative refinement, emerges as the preferred choice for achieving long-term optimality and compliance in facility location planning.

Author Contributions:

Conceptualization, EA.; Methodology, EA. and GM.; Software, EA.; Validation, EA., GM., CDN., and ABR.; Forma Analysis, EA.; Investigation, EA. and CDN.; Investigation, EA.; Resources, EA., CDN., GM. and ABR.; Data Curation, EA.; Writing-Original Draft Preparation, EA.; Writing-Review and Editing, EA., CDN. and ABR.; Visualisation, EA.; Supervision, CDN. and ABR.; Project Administration, EA.; Funding Acquisition, No Funding. All authors have read and agreed to the published version of the manuscript.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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