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Grow proof ability student through mathematical working space strategies

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Abstract: This research aims to assist students in solving mathematical proof problems. This qualitative study investigates the proving ability of mathematics education students in the Real Analysis course, with a special focus on sequence material. The research design incorporates experiments to evaluate the effectiveness of the strategies used. Participants are students who take part in the Real Analysis program at private universities in the northern coastal region of western Central Java, Indonesia, specifically in Tegal City during the 2022/2023 academic year. Three study participants were selected, each representing a different level of evidentiary ability. Data is collected through work documents, interviews, and structured testing, and then analyzed using iterative techniques, including data condensation, data exposure, and verification. The findings show that most students can follow the evidence and correct their mistakes. In the future, this research will prioritize developments related to proof to improve quality-proof capabilities and explore the obstacles faced in studying proof problems. This research can ultimately contribute to more effective teaching of Real Analysis by providing a deeper understanding of the evidence problem that can be easily learned.
Keywords: *Mathematical working space strategy, Model guided discovery learning, Proof ability, Real analysis.*

1. Introduction

The ability to process evidence is essential for mathematics education students, as it directly influences their proficiency in developing working memory and cognition, both of which impact their ability to construct proofs [1]. Additionally, a solid understanding of basic mathematical concepts is required [2]. The creation, construction, and communication of mathematical knowledge heavily rely on evidence, enabling mathematicians to persuade others of the correctness of their ideas and statements [1, 3]. Further emphasizes that a strong grasp of mathematical concepts is crucial for building robust evidence.

The process of discovering, verifying, explaining, systematizing, and communicating mathematical knowledge relies heavily on evidence. Students must be able to provide reasonable justifications and demonstrate that mathematical statements are correct. Proofs can lead to new insights, discoveries, and developments [4, 5]. The Real Analysis course in the Department of Mathematics and Mathematics Education is rich in evidentiary content. Real Analysis is a complex subject that requires deep thinking. Building and understanding evidence necessitates a strong grasp of mathematical concepts. Before studying Real Analysis, students must be prepared to comprehend the evidence presented. Additionally, they must develop an understanding of proof and enhance their problem-solving skills. Both of these abilities can be cultivated through the practice of solving problems related to proofs [6].

Most students of mathematics and mathematics education consider Real Analysis to be a very difficult subject. Many students are unfamiliar with the concept of proof and struggle to understand it. To enhance students' ability to comprehend and construct evidence, they must develop confidence in their reasoning skills. Unfortunately, many college students possess a fixed mindset when it comes to learning mathematics, which leads to a lack of confidence in understanding and constructing evidence

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[5]. The epistemology of students in the Real Analysis course reveals several challenges they face when studying proofs. These challenges include difficulties in understanding concepts (definitions), uncertainty about how to begin constructing proofs, lack of knowledge regarding the application of known definitions and principles, and confusion about what needs to be proven [7, 8].

Students often struggle to understand the role and function of proof in mathematics, both in the short and long term. Evidence is crucial for verifying the correctness of statements, providing logical arguments, constructing new mathematics, and communicating mathematical concepts. It also plays a vital role in systematizing statements within axiomatic systems [9]. The application of evidence in mathematics education is essential because it enhances critical thinking and problem-solving skills[10]. The imparting of ideas, the development and communication of knowledge, and the contribution to the development of mathematics are all significant aspects of proof. By emphasizing these aspects, educators can better support students in their understanding and application of mathematical proofs [11].

1.1. The Needs of Students

The Guided Discovery Learning learning model has a positive impact on comprehension skills and mathematical skills, while the benefits of the Guided Discovery Learning Model on mathematics learning are: (1) Improvement of Conceptual Comprehension: Research conducted by Dumitraşcu [12] shows that the integration of Guided Discovery Learning in real analysis teaching allows students to understand concepts more deeply compared to conventional approaches. (2). Development of Critical Thinking Skills: A study by Kariman, et al. [13] confirms that the Guided Discovery Learning model, when combined with argument mapping, can improve students' critical thinking skills in understanding the changes that occur in the environment. (3). Improvement of Practice Skills: Aagesen, et al. [14] found that the application of Guided Discovery Learning in surgical skills courses was more effective in improving students' understanding and practical skills compared to traditional teaching methods. (4). Positive Impact in Mathematics Education: Kariman, et al. [13] shows that modules based on Guided Discovery Learning are able to improve students' understanding of concepts and critical thinking skills in the field of mathematics. (5). Technology Support for Interactive Learning: Kniha, et al. [15] reveals that the combination of Guided Discovery Learning and technology, such as video-based teaching, results in better outcomes in oral surgical skills training. (6). Increased Interest and Motivation: Studies by Nurhayani, et al. [16] show that the application of Guided Discovery Learning can increase students' interest in learning mathematics and encourage them to be more active in the learning process.

Next for the concept of Mathematical Working Space is a theoretical and methodological model applied in mathematics education research to analyze and understand mathematical activities carried out by students and teachers. This model integrates the epistemological and cognitive aspects of mathematical activities, which are organized in two main domains: the epistemological domain and the cognitive domain [17-19].

The Mathematical Working Space is characterized by three interconnected dimensions, which are crucial for carrying out thorough mathematical activities:Semiotic Dimension: Refers to the use of signs and symbols in mathematical reasoning and communication processes [19, 20]. instrumental Dimension: Involves tools and instruments, both physical and digital, used in the execution of mathematical tasks [17]. Discursive Dimension: Deals with the language and discourse applied in explanations and mathematical arguments [19].

Mathematical Woking Space has the following benefits: (1). Holistic Understanding: By integrating epistemological and cognitive aspects, Mathematical Woking Space provides a comprehensive framework for understanding mathematical activities [19, 20]. (2). Identification of Misunderstandings: This model serves to identify and address misunderstandings that may not be directly detected by teachers [17]. (3). Improvement of Teaching Practices: Mathematical Woking Space offers a structured approach to improving teaching practices by emphasizing the interaction between different dimensions in mathematics activities [18].

Based on the above proof problem, students need a supportive and inclusive learning environment to enhance their evidentiary skills. A well-designed learning setting encourages students to identify patterns, ask questions, and seek help when necessary. Additionally, prior knowledge significantly impacts student learning in the classroom Baki [21]. A caring, consistent, and inclusive environment is essential for students to overcome difficulties, take risks, speak up, and ask for help."

The chosen learning model must meet the needs and conditions of students [22]. A learning model known as guided discovery learning encourages students to investigate and develop their concepts. This model allows students to enhance their critical problem-solving and critical thinking skills, with lecturers serving as facilitators who analyze the challenges students face to help them solve problems [14-16, 23].

Strategies are essential because they help students manage their thoughts during evidentiary activities. To maximize the impact on learning objectives, evidence-based strategies must be carefully selected [24]. This study employs the Mathematical Working Space strategy, which incorporates semiotics to connect representations that visualize mathematical problems. The instrumental origin represents the stage where the knowledge gathered can be utilized to construct evidence. When this evidence is validated, it is referred to as genesis [17, 25, 26].

The Discussion Material Worksheet serves as a vital learning tool in this study. This worksheet employs the Mathematical Working Space strategy, which includes Sequence material [27]. The content utilized in this paper is derived from comprehensive Semiotic sources [28]. The worksheet encompasses the background, core concepts, and attributes of each available definition. Subsequently, through group discussions, students are required to interpret each definition in language that is easy to understand. Furthermore, every lemma, theorem, or proof problem incorporates a semiotic, instrumental, and discursive foundation. The Discussion Material Worksheet is specifically designed to aid students in learning Real Analysis.

Mathematical research related to proof and learning using the Mathematical Working Space strategy has demonstrated success. For instance, found positive outcomes in evidentiary problems in Geometry Gómez-Chacón and Kuzniak [18] applied this strategy in evidentiary research on functional problems Minh and Lagrange [29] and reported success in the study of histogram problems Derouet and Parzysz [28] noting that its application helps students improve their creative reasoning and successfully solve mathematical problems [28, 30-34]. Therefore, it is evident that there are distinctions between previous research and the current study, particularly as this research examines the evidentiary process in Real Analysis through the stages of visualization, construction, and validation

To optimize the development of proof skills in Real Analysis lectures, research must be conducted. Additionally, to assess the ability of prospective mathematics teachers to solve mathematical proof problems, qualitative research was carried out involving several mathematics teacher candidates. This research aims to enhance the mathematical proof abilities of prospective mathematics teachers in completing the process of proving Sequence material within the Real Analysis course by utilizing the Guided Discovery learning model and the Mathematical Working Space strategy.

2. Method

2.1. Research Design

A qualitative approach is employed in this study to examine the ability of prospective mathematics teachers to solve proof problems related to Sequence material in the Real Analysis course. An experimental design is utilized for the research. The focus of this study is on three subjects who are considered to meet the objectives of the research.

2.2. Research Subject

This research involved 27 Strata One mathematics education students who were enrolled in the Real Analysis course. After the students learned the material in Sequence for six meetings, a test was conducted in the seventh meeting to measure their proof abilities. Based on the test results, the students

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were categorized into three groups: very good (L), medium (M), and poor (S). Three students from each group were randomly selected. The lecturer then recommended five students out of the nine selected, but only three L_2, M_5, and S_3 students were willing to take part in the research.

2.3. Data Collection

Data on mathematical proof ability were collected through semi-structured tests and interviews. The test consists of six descriptive questions that address the following topics: the limit of a sequence, the squeeze theorem, sequence ratios, monotone sequences, subsequences, and Cauchy sequences. Then three subjects were selected for the interview.

Tests were administered to assess students' abilities in completing the evidentiary process. The criteria for evaluating students' abilities in solving evidentiary problems are presented in Table 1 below.

Table 1.
Criteria for Proof.

Number	Rumus	Criterion
1.	$x \geq \bar{x} + d.s$	Tall (L)
2	$\bar{x} - s.d \leq x < \bar{x} + d.s$	Keep (M)
3	$x \leq \bar{x} - d.s$	Low (S)

During the interview, students were presented with problems related to the sequence material and its connection to the proof process. The purpose of this interview method is to gain deeper insights into the students' proof processes when solving evidentiary problems.

The following indicators are used to measure the ability in mathematical proof: 1) Reading of evidence with its evidentiary aspects, which includes making hypotheses (conjectures) based on the patterns and characteristics of several statements, as well as proving the obtained conjectures through deduction; 2) Evidence construction with its evidentiary aspect, which involves applying definitions and related characteristics, sequencing the evidentiary and construction steps into formal evidence, and demonstrating the ability to use premises, definitions, or theorems associated with the statement to build valid evidence.

2.4. Analyzing of Data

To analyze the test and interview data, with interactive techniques, namely data condensation, exposure, and verification. The purpose of this study is to improve students' ability in the mathematical proof process by helping to solve the problems they write for analysis and then analyzing indicators that show proof ability. It is hoped that the new findings will result in practical, measurable, and applicable measures that can contribute to mathematics education.

3. Results and Discussion

3.1. Implementation

Students in the Mathematics Education study program at Pancasakti Tegal University use the Guided Discovery Learning model in conjunction with the Mathematical Working Space strategy to engage in the Real Analysis of Sequence material. The students do not feel burdened by the learning model used during lectures. To solve the problem of proof, students can investigate the findings and seek assistance from the teacher if needed to work through the mathematical proof they are faced with.

In addition, the use of the Mathematical Working Space strategy for discussion material is very helpful for students in addressing proving problems in Real Analysis. The worksheet for the discussion material on sequences contains definitions, theorems, and proof questions. Each definition is accompanied by attributes that are useful for sequencing the evidentiary steps, which greatly assists students in their proof work. Furthermore, for every definition, theorem, lemma, or proof problem, students must be able to interpret it in language that is easy to understand and conveys a clear meaning.

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This interpretation can be facilitated through discussions within their groups. Furthermore, each theorem, lemma, and proof question are accompanied by a background, premise, and conclusion. The premise serves as the first step in the proof and should lead to the conclusion as the result. Similarly, the construction of evidence should be supported by relevant facts and should sequence the available evidentiary steps to obtain a valid construction. After that, the premises, definitions, theorems, or problems that have been studied about the statement can be used to build the evidence construction. It is also advisable to conduct a preliminary analysis when solving evidentiary problems to avoid mistakes in gathering the facts to be used.

The results of the interviews conducted with students after they completed the Real Analysis lesson in this study are as follows:

T: What are your thoughts on learning in this sequence of materials?

L_2: I enjoy using this learning approach because it helps me complete this worksheet and other tasks effectively and on time.

M_5: I find it very helpful because it used to be quite difficult for me to study the problem of proof. However, now I am better at completing it, thanks to the guidance of my lecturers, who are always watching over me.

S_3: Fear and laziness, which often arose when studying the problem of proof, are now gradually disappearing. Understanding evidence learning today is easier than ever.

T: What are your thoughts on the strategies used to learn Real Analysis?

L_2: I think it's fun and easy to use for other proof materials.

M_5: I will use this strategy when necessary so that I can master the proof correctly.

S_3: This strategy has opened my eyes to the concept of proof.

Based on the results of the interview, it can be seen that L_2 students, who previously almost understood mathematical proof problems, have become even better. M_5 students, who were previously still confused about proof problems, have also improved. Additionally, S_3 students, who were previously still afraid of proof problems, now feel comfortable learning about proof.

3.2. Troubleshooting

Based on the two completion indicators used and the six available questions. The three students can complete the proof correctly on the Sequence Ratio problem because each of them can perform the initial analysis correctly, thereby enabling them to proceed with a simple valid proof. In the problem of the Squeeze Theorem, L_2 and M_5 students can solve this simple proof because they can correctly determine the premise, making it easier for them to apply it. Additionally, L_2 and M_5 students can provide valid examples of divergent Sequences because they both understand the definition of convergent Sequences well. Meanwhile, only L_2 students can solve the problem of Cauchy Sequence and Sequence limits. L_2 students can show the final results of the proof in these two questions. The student conducted an initial analysis to obtain the final result and then organized the proof steps to achieve a valid outcome. While M_5 students only possess the ability to conduct a preliminary analysis on the Sequence limit problem, their proof is not constructive. In the case of the Monotone Sequence Problem, there is no single criterion that can complement valid evidence. Only L_2 students can perform the initial analysis correctly; however, they were not careful when executing the evidentiary steps, which prevented them from arriving at correct conclusions.

The test conducted on the Sequence material is designed to assess students' mathematical proof abilities in solving problems. Students are required to answer six evidence-oriented questions. The focus of the test analysis is on indicators of the student's mathematical proof performance, as shown in Table 2

Table 2.
Evidentiary Ability and Measured Aspect.

Number	Indicators	Measured Aspects	Sub- Materials	L_2	M_5	S_3
1.	Reading Evidence	Make a hypothesis (conjecture) based on the pattern and nature of several statements and prove the obtained conjecture by deduction.	Squeeze Theorem	√	√	×
			Cauchy Sequence	√	×	×
2.	Constructing Evidence	Ability to apply definitions and related properties and sequence the steps of proof and construction into formal evidence.	Sequence Limit	√	×	×
			Sequence Ratio	√	√	√
		Ability to use premises, definitions, or statement-related theorems to build a proof	Monoton Sequence	×	×	×
			Subsequences	√	√	×

Based on the results of solving the questions above, it was found that L_2 students already understood the concept of proof, grasped the flow of proof, and could draw valid conclusions. Meanwhile, S_3 students do not thoroughly understand the concept of proof and cannot follow the evidence that has been examined.

3.3. *Results of Evidentiary Ability*

The evidentiary test was conducted at the seventh meeting after the sequence material was completed. Table 1 shows how to calculate the evidentiary criteria based on the test results, while Table 3 presents the results of the evidentiary criteria, namely:

Table 3.
Results of Evidentiary Criteria

Number	Types of categories	A large number of students
1.	The upper category (L)	16
2.	The middle category (M)	6
3.	The lower category (S)	3

Observations made in each category showed that high-achieving students enjoyed using the newly learned lessons, and the Discussion Material worksheets made learning about evidence easier. Students engage in group discussions on this worksheet, allowing them to grasp the concept of proof more easily. If students read the evidence correctly and carefully, they can understand it better. They can easily and accurately interpret all the definitions, theorems, lemmas, and evidentiary problems presented in the worksheet. Additionally, students can use the strategies provided in the worksheet to construct evidence. Students already understand the concepts, definitions, and theorems, allowing them to comprehend the steps of proof correctly. Most students can identify and correct symbols, statements, and premises in incorrect proof steps or provide reasons for these errors. They can also apply the evidentiary steps to similar statements. Furthermore, students successfully use relevant premises, definitions, or theorems to construct evidence. Students in this class can understand and follow evidence effectively. Finally, students with high grades can easily provide accurate and valid conclusions. The results of the work on the Cauchy Sequence carried out by L_2 are as follows:

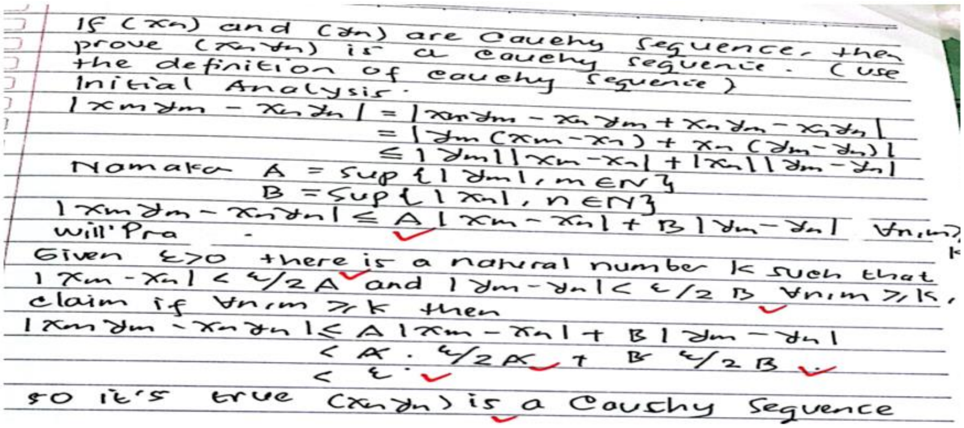
If (x_n) and (y_n) are Cauchy sequence, then prove $(x_n y_n)$ is a Cauchy sequence. (use the definition of cauchy sequence)
Initial Analysis:
$|x_m y_m - x_n y_n| = |x_m y_m - x_n y_m + x_n y_m - x_n y_n|$
$= |y_m (x_m - x_n) + x_n (y_m - y_n)|$
$\leq |y_m||x_m - x_n| + |x_n||y_m - y_n|$
Nomaka $A = \sup\{|y_m|, m \in N\}$
$B = \sup\{|x_n|, n \in N\}$
$|x_m y_m - x_n y_n| \leq A|x_m - x_n| + B|y_m - y_n| \quad \forall n, m$
will'Pra
Given $\varepsilon > 0$ there is a natural number K such that $|x_m - x_n| < \varepsilon/2A$ and $|y_m - y_n| < \varepsilon/2B \quad \forall n, m \geq K$,
claim if $\forall n, m \geq K$ then
$|x_m y_m - x_n y_n| \leq A|x_m - x_n| + B|y_m - y_n|$
$< A \cdot \varepsilon/2A + B\, \varepsilon/2B$
$< \varepsilon$.
so it's true $(x_n y_n)$ is a Cauchy Sequence

Figure 1.
High category performance result (L2).

The results of the lecturer's (T) interview with the students (L_2) are as follows:

T: What do you think of this material sequence during the lecture?

L_2: Increase knowledge of the problem of proof.

T: How to complete the evidence related to the problem of proof?

L_2: Before embarking on proving a theorem, it is indispensable to thoroughly comprehend the underlying problem and then do the Initial Analysis

T: Why do you need to understand the meaning of the problem?

L_2: Understanding the problem makes it easy to identify the premise and conclude what needs to be proven. With clarity about these points, you can then take the required evidentiary steps to arrive at your conclusion. Each step builds upon the last, creating a logical sequence that leads directly to your desired outcome. Thus, clear comprehension simplifies the process of constructing a valid proof.

T: What is the need for an initial analysis

L_2: To facilitate proof, in this row problem, the theories used to prove convergent rows are numerous so that by conducting a preliminary analysis, we can determine the right theory that corresponds to the characteristics of the problem. To do this, do a preliminary analysis first, as is done in the Cauchy row problem, so that it is easy to determine the value. $\frac{\varepsilon}{2A}$ *and* $\frac{\varepsilon}{2B}$ for each row, so that if the two rows are added together it will yield ε.

Students in the middle category (M) have expressed their satisfaction with the newly implemented learning model, although they still request continuous support from their lecturers. Their understanding of the steps involved in proof and the application of definitions for proving concepts has notably improved. Below is one of the tests works on Cauchy Sequence problems completed by a student in the intermediate category, identified as M_5. This work reflects their enhanced grasp of the material and demonstrates their ability to apply the concepts learned effectively.

The results indicated that students in the middle category (M) expressed satisfaction with the new learning model. However, they emphasized the need for ongoing support from their lecturers to enhance their learning experience. These students reported gaining a deeper understanding of the proof process and the application of definitions in mathematical proofs. An example of an exam question related to the Cauchy Sequence, designed for students in the intermediate category, is provided below.

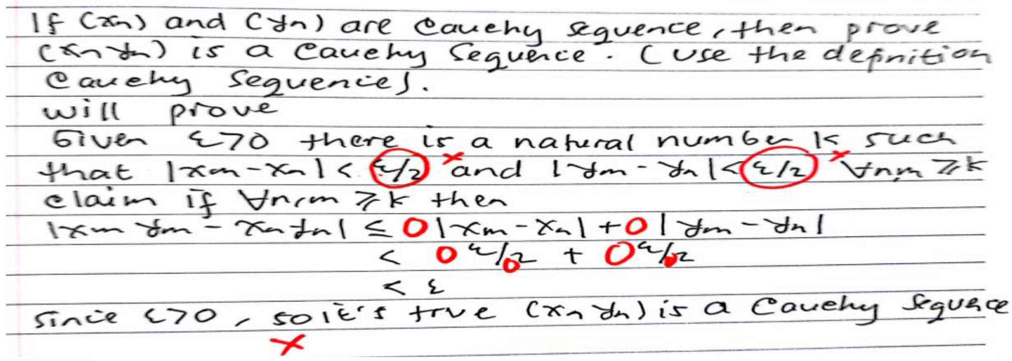

Figure 2.
Medium category test answer result (M_5)

The results of the interviews conducted by M_5 lecturers and students revealed that students in category M still experience confusion when verifying mathematical statements to determine their validity. Although they have learned various methods for constructing valid proofs, they struggle with how to appropriately utilize premises, definitions, or statements related to theorems in their reasoning. These students can follow the provided evidence but find it challenging to articulate how, why, or where any discrepancies in the proof may exist.

T: What do you think of the new atmosphere presented in this sequence material?

M_5: It is better than ever

T: What do you think of the use of worksheets on sequence material?

M_5: I find it relatively easy to understand, despite needing some effort at times. Nonetheless, I regularly seek clarification from our instructor.

T: What causes difficulty for you in solving the evidence questions involving Cauchy Sequences?

M_5: I don't perform the initial analysis, so when the proof fails to determine the value for every sequence ε

Students from the lower category (S) appreciated the new learning approach. However, this group takes longer to comprehend the worksheets compared to students from the other two categories. This type of understanding of valid proofs requires repeated reinforcement.

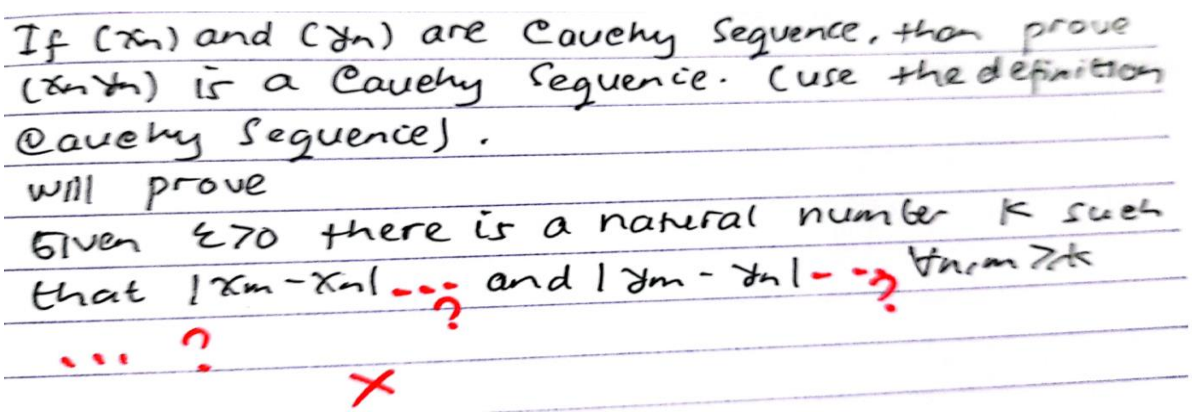

Figure 3.
Low category test answer (S_3).

The findings from the interviews conducted with lecturers and S_3 students indicate that understanding the nature of certain statements and hypotheses (conjectures) based on patterns, as well

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as proving these hypotheses deductively, often proves challenging. Students are hesitant to organize the evidentiary steps and convert them into formal proofs using relevant definitions or characteristics. This difficulty is compounded for students in category S, who can only grasp a small portion of the evidence being studied, making it hard for them to initiate the evaluation process.

T: Are you happy when you sit in a group and discuss the evidence with your friends

S_3: Very happy

T: What do you think about using discussion material worksheets with the Mathematical Working Space strategy while focusing on Sequence material?

S_3: While I remain somewhat confused, this experience has increased my enthusiasm for learning

T: What causes you to be unable to solve the Cauchy sequence problem correctly?

S_3: I forgot the definition of the Cauchy Sequence.

The interview indicates that the evidentiary indicators are not being prioritized by S_3.

4. Discussion

One of the benefits of the Guided Discovery learning model is that it transforms the student learning environment into a more active setting, encouraging students to collaborate and engage in problem-solving activities, particularly in Real Analysis. This outcome aligns with the findings of numerous studies that employ this model in exact sciences, demonstrating its efficacy in enhancing student participation and improving learning outcomes. Similarly, applying this model to teach Real Analysis fosters greater student engagement in evidentiary learning. Studies have shown that students who use the Guided Discovery Learning model in Real Analysis achieve a successful comprehension of complex concepts [12]. Despite feeling confused when learning real analysis, the worksheet discussion material has been prepared with the Mathematical Working Space strategy, thereby alleviating the burden. Additionally, statements indeed require guidance to make learning both meaningful and enjoyable [35]. This discussion material worksheet is structured with a mathematical workspace strategy that aids students in mastering Real Analysis [36-38]. By helping students understand definitions, theorems, lemmas, and related problems in simple language, these worksheets facilitate the development and validation of evidence. Therefore, utilizing this method enhances the overall learning experience for students engaged in Real Analysis.

Understanding proofs is crucial for demonstrating theorems and solving proof problems; therefore, all students must grasp the concept of definitions. Various approaches can enhance students' thinking by encouraging them to write valid proofs [39]. By employing the Mathematical Working Space strategy, students can more easily comprehend the definitions presented in the discussion material worksheets. Every student should be familiar with the background, meaning, and characteristics of definitions. This knowledge will facilitate their ability to prove theorems and solve proof problems, making the process significantly easier and more manageable.

During lectures, students often engage in group discussions to answer questions, which is very helpful in finding solutions [13]. The Guided Discovery learning model is particularly effective in real-analysis education [12, 39]. At the end of each session, students work individually on evidence-related questions, which encourages them to enhance their proof skills. Additionally, the discussion material worksheets have significantly aided students in learning Real Analysis and overcoming various learning difficulties. Worksheets that incorporate mathematical workspace strategies are especially beneficial for understanding proofs [17, 19, 26, 40, 41].

The use of discussion material worksheets significantly aids students in mastering Real Analysis. Initially, students often struggle with proving concepts they encounter; however, flexible worksheets can help mitigate these challenges. Many students report that these worksheets are instrumental in analyzing mathematical problems. Once students have a solid grasp of the fundamentals of analysis and proof techniques, they find it easier to tackle more complex mathematical problems. This indicates that mastering proof concepts lays a strong foundation for further learning [42-44]. Support this notion, suggesting that once students understand the concept of proof, they are better equipped to demonstrate their knowledge effectively [45].

One of the findings is that students are motivated to tackle proof questions after implementing the strategies used. The lecturer incorporated several proof questions to ensure that students do not feel overwhelmed by their assignments. Research by Aisyah, et al. [46] found that when teachers frequently provide proof questions, it allows students to practice their reasoning skills. This practice positively impacts students, as they become accustomed to solving problems related to proofs.

5. Conclusion

In Real Analysis learning, the Guided Discovery learning model is highly effective because it encourages students to approach problems as proofs. By utilizing the Mathematical Working Space strategy, discussion material worksheets can significantly aid students in understanding proof concepts. This effectiveness stems from the fact that the worksheets incorporate visualization, construction, and validation elements, which enhance the learning experience.

The Mathematical Working Space strategy facilitates representation, visualization, and construction to prove or validate evidence. This visualization helps students better understand concepts and definitions. Additionally, it enables students to use definitions as a foundation for providing reasoning behind the correct proof steps or for correcting symbols, statements, and premises in the case of incorrect proof steps.

Students possess a thorough comprehension of definitions, enabling them to analyze mathematical statements and determine whether they are true or false through the application of example rejections. They also excel at organizing and rearranging factual information while sequencing the steps required to construct valid arguments. Furthermore, students can connect established knowledge regarding the statement to the claims needing verification. To build robust evidence, these facts can be integrated into existing definitions, theorems, or previously proven assertions related to the statement.

Discussion can be an effective strategy to encourage democratic habits, increase student enthusiasm, and enhance intellectual intelligence. After learning Real Analysis through worksheets that utilize the Mathematical Working Space strategy, students demonstrate several competencies: they can understand general statements and test them with examples; identify errors in proofs and correct them; and draw accurate conclusions. The implementation of evidence-based strategies, such as visualization, discussion, and manipulation, significantly contributes to improving the learning experience in Real Analysis.

This study focuses on Real Analysis, specifically the material related to sequences. The primary objective is to prove the convergence of sequences. Bartle's approach utilizes a variety of theorems and definitions for this purpose, with each session dedicated to discussing theorems that help establish the convergence of sequences. As a result, students gain a clear understanding and can effectively apply these theorems to solve problems. After completing the material, students take a test; however, they often find it challenging to determine which theorem applies to specific problems. It is essential for students to correctly utilize theorems and definitions relevant to the problems they encounter. To address this issue, a discussion material worksheet was developed using the Mathematical Working Space strategy. The implementation of these worksheets enhances student engagement in learning Real Analysis, encouraging lecturers to pose questions that require evidence-based reasoning. Students benefit from applying the strategies outlined in these worksheets, as they assist in tackling proof-related problems.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

References

[1] H. Rocha, "Mathematical proof: From mathematics to school mathematics," *Philosophical Transactions of the Royal Society A*, vol. 377, no. 2140, p. 20180045, 2019. https://doi.org/10.1098/rsta.2018.0045
[2] E. Bettinger, S. Ludvigsen, M. Rege, I. F. Solli, and D. Yeager, "Increasing perseverance in math: Evidence from a field experiment in Norway," *Journal of Economic Behavior & Organization*, vol. 146, pp. 1-15, 2018. https://doi.org/10.1016/j.jebo.2017.11.032
[3] L. Cragg and C. Gilmore, "Skills underlying mathematics: The role of executive function in the development of mathematics proficiency," *Trends in Neuroscience and Education*, vol. 3, no. 2, pp. 63-68, 2014. https://doi.org/10.1016/j.tine.2013.12.001
[4] T. CadwalladerOlsker, "What do we mean by mathematical proof?," *Journal of Humanistic Mathematics*, vol. 1, no. 1, pp. 33-60, 2011. https://doi.org/10.5642/jhummath.201101.04
[5] D. A. Stylianou, M. L. Blanton, and O. Rotou, "Undergraduate students' understanding of proof: Relationships between proof conceptions, beliefs, and classroom experiences with learning proof," *International Journal of Research in Undergraduate Mathematics Education*, vol. 1, pp. 91-134, 2015. https://doi.org/10.1007/s40753-015-0003-0
[6] T. B. L. Tran, T. N. Ho, S. V. Mackenzie, and L. K. Le, "Developing assessment criteria of a lesson for creativity to promote teaching for creativity," *Thinking Skills and Creativity*, vol. 25, pp. 10-26, 2017. https://doi.org/10.1016/j.tsc.2017.05.006
[7] H. B. Chand, "Difficulties experienced by undergraduate students in proving theorems of real analysis," *Scholars' Journal*, pp. 149-163, 2021. https://doi.org/10.3126/scholars.v4i1.42475
[8] I. Widiati and A. Sthephani, "Difficulties analysis of mathematics education students on the real analysis subject," presented at the In Journal of Physics: Conference Series (Vol. 1088, No. 1, p. 012037). IOP Publishing, 2018.
[9] A. Bordg, "A Replication crisis in mathematics?," *The mathematical intelligencer*, pp. 1-5, 2021. https://doi.org/10.1007/s00283-020-10037-7
[10] G. Güler, "The difficulties experienced in teaching proof to prospective mathematics teachers: Academician views," *Higher Education Studies*, vol. 6, no. 1, pp. 145-158, 2016. https://doi.org/10.5539/hrs.v6n1p145
[11] V. Durand-Guerrier, P. Boero, N. Douek, S. S. Epp, and D. Tanguay, "Examining the role of logic in teaching proof," vol. 15, 2012. https://doi.org/10.1007/978-94-007-2129-6_16
[12] D. Dumitraşcu, "Integration of guided discovery in the teaching of real analysis," *Primus*, vol. 19, no. 4, pp. 370-380, 2009. https://doi.org/10.1080/10511970802072368
[13] D. Kariman, Y. Harisman, A. Sovia, and R. C. I. Prahmana, "Effectiveness of guided discovery-based module: A case study in padang city, Indonesia," *Journal on Mathematics Education*, vol. 10, no. 2, pp. 239-250, 2019. https://doi.org/10.22342/jme.10.2.6610.239-250
[14] A. H. Aagesen *et al.*, "The benefits of tying yourself in knots: Unraveling the learning mechanisms of guided discovery learning in an open surgical skills course," *Academic Medicine*, vol. 95, no. 11S, pp. S37-S43, 2020. https://doi.org/10.1097/ACM.0000000000003646
[15] K. Kniha *et al.*, "Guided discovery learning: A follow-up study of try-it-yourself surgery and subsequent video-assisted teaching for oral surgical skills training," *European Journal of Dental Education*, vol. 27, no. 1, pp. 29-35, 2023. https://doi.org/10.1111/eje.12772
[16] N. Nurhayani, R. Rosnawati, and T. Amimah, "Optimization of guided discovery learning models to increase students' interest in mathematics," *Infin. J*, vol. 9, no. 1, p. 69, 2020. https://doi.org/10.22460/infinity.v9i1.p69-80
[17] A. Kuzniak and A. Nechache, "On forms of geometric work: A study with pre-service teachers based on the theory of Mathematical Working Spaces," *Educational Studies in Mathematics*, vol. 106, no. 2, pp. 271-289, 2021. https://doi.org/10.1007/s10649-020-10011-2
[18] I. M. Gómez-Chacón and A. Kuzniak, "Spaces for geometric work: figural, instrumental, and discursive geneses of reasoning in a technological environment," *International Journal of Science and Mathematics Education*, vol. 13, pp. 201-226, 2015. https://doi.org/10.1007/s10763-013-9462-4
[19] E. M. Delgadillo and L. Vivier, "Mathematical working space and paradigms as an analysis tool for the teaching and learning of analysis," *ZDM*, vol. 48, pp. 739-754, 2016. https://doi.org/10.1007/s11858-016-0777-9
[20] A. Kuzniak, D. Tanguay, and I. Elia, "Mathematical Working Spaces in schooling: an introduction," *ZDM*, vol. 48, pp. 721-737, 2016.
[21] M. Baki, "The development of mathematical knowledge for teaching of mathematics teachers in lesson analysis process," *European Journal of Educational Research*, vol. 5, no. 4, pp. 165-171, 2016. https://doi.org/10.12973/eu-jer.5.4.165

[22] W. Winarso and A. A. Haqq, "Where exactly for enhance critical and creative thinking: The use of problem posing or contextual learning," *European Journal of Educational Research*, vol. 9, no. 2, pp. 877-887, 2020. https://doi.org/10.12973/eu-jer.9.2.877

[23] Y. Yerimadesi *et al.*, "Guided discovery learning-based chemistry e-module and its effect on students' higher-order thinking skills," *Jurnal Pendidikan IPA Indonesia*, vol. 12, no. 1, pp. 168-177, 2023. https://doi.org/10.15294/jpii.v12i1.42130

[24] R. W. Y. Putra *et al.*, "Systematic literature review on the recent three-year trend mathematical representation ability in scopus database," *Infinity Journal*, vol. 12, no. 2, pp. 243-260, 2023. https://doi.org/10.22460/infinity.v12i2.p243-260

[25] L. M. Angraini and A. Wahyuni, "The effect of concept attainment model on mathematical critical thinking ability," *International Journal of Instruction*, vol. 14, no. 1, pp. 727-742, 2021.

[26] I. Elia, S. Özel, A. Gagatsis, A. Panaoura, and Z. E. Y. Özel, "Students' mathematical work on absolute value: focusing on conceptions, errors and obstacles," *ZDM*, vol. 48, no. 6, pp. 895-907, 2016.

[27] R. G. Bartle and D. R. Sherbert, *Introduction to real analysis*. John Wiley & Sons, Inc., 2000.

[28] C. Derouet and B. Parzysz, "How can histograms be useful for introducing continuous probability distributions?," *ZDM*, vol. 48, no. 6, pp. 757-773, 2016.

[29] T. K. Minh and J.-B. Lagrange, "Connected functional working spaces: A framework for the teaching and learning of functions at upper secondary level," *ZDM*, vol. 48, no. 6, pp. 793-807, 2016. https://doi.org/10.1007/s11858-016-0774-z

[30] A. Bacelo and I. M. Gomez-Chacon, "Characterising algorithmic thinking: A university study of unplugged activities☆," *Thinking Skills and Creativity*, vol. 48, p. 101284, 2023. https://doi.org/10.1016/j.tsc.2023.101284

[31] T. Bauer, R. Biehler, and E. Lankeit, "ConcepTests in undergraduate Real Analysis: Comparing peer discussion and instructional explanation settings," *International Journal of Research in Undergraduate Mathematics Education*, vol. 9, no. 2, pp. 426-460, 2023. https://doi.org/10.1007/s40753-022-00167-y

[32] M.-H. Shih and W.-P. Sung, "Shaking table test and verification of development of an accumulated semi-active hydraulic damper as an active interaction control device," *Sādhanā*, vol. 41, pp. 1425-1442, 2016. https://doi.org/10.1007/s12046-016-0570-z

[33] Z. K. Szabo, P. Körtesi, J. Guncaga, D. Szabo, and R. Neag, "Examples of problem-solving strategies in mathematics education supporting the sustainability of 21st-century skills," *Sustainability*, vol. 12, no. 23, p. 10113, 2020. https://doi.org/10.3390/su122310113

[34] J. Lithner, "Principles for designing mathematical tasks that enhance imitative and creative reasoning," *Zdm*, vol. 49, no. 6, pp. 937-949, 2017. https://doi.org/10.1007/s11858-017-0867-3

[35] K. Weber, "Effective proof reading strategies for comprehending mathematical proofs," *International Journal of Research in Undergraduate Mathematics Education*, vol. 1, pp. 289-314, 2015. https://doi.org/10.1007/s40753-015-0011-0

[36] Y. Zengin, "Development of mathematical connection skills in a dynamic learning environment," *Education and Information Technologies*, vol. 24, no. 3, pp. 2175-2194, 2019. https://doi.org/10.1007/s10639-019-09870-x

[37] A. T. Ottenbreit-Leftwich, K. D. Glazewski, T. J. Newby, and P. A. Ertmer, "Teacher value beliefs associated with using technology: Addressing professional and student needs," *Computers & Education*, vol. 55, no. 3, pp. 1321-1335, 2010.

[38] M. K. Stein, J. L. Russell, V. Bill, R. Correnti, and L. Speranzo, "Coach learning to help teachers learn to enact conceptually rich, student-focused mathematics lessons," *Journal of Mathematics Teacher Education*, pp. 1-26, 2022.

[39] A. Stefanowicz, J. Kyle, and M. Grove, "Proofs and mathematical reasoning university of birmingham," ed: September, 2014.

[40] M. Rodd, "Transition from "It looks like" to "It has to be" in a geometric workspace: Affect and attention," *Bolema: Boletim de Educação Matemática*, vol. 30, pp. 142-164, 2016. https://doi.org/10.1590/1980-4415v30n54a07

[41] M. Santos-Trigo, L. Moreno-Armella, and M. Camacho-Machín, "Problem solving and the use of digital technologies within the mathematical working space framework," *ZDM*, vol. 48, pp. 827-842, 2016. https://doi.org/10.1007/s11858-016-0757-0

[42] M. Omar, G. Karakok, M. Savic, H. E. Turkey, and G. Tang, "I felt like a mathematician: Problems and assessment to promote creative effort," *Primus*, vol. 29, no. 1, pp. 82-102, 2019.

[43] G. Hanna and E. Barbeau, "Proofs as bearers of mathematical knowledge," *ZDM - Int. J. Math. Educ*, vol. 40, no. 3, pp. 345–353, 2008. https://doi.org/10.1007/s11858-008-0080-5

[44] P. Regier and M. Savic, "How teaching to foster mathematical creativity may impact student self-efficacy for proving," *The Journal of Mathematical Behavior*, vol. 57, p. 100720, 2020.

[45] M. Nadlifah and S. Prabawanto, "Mathematical proof construction: Students' ability in higher education," in *Journal of Physics: Conference Series*, 2017, vol. 895, no. 1, p. 012094.

[46] N. Aisyah, E. Susanti, M. Meryansumayeka, T. Y. E. Siswono, and S. M. Maat, "Proving geometry theorems: Student prospective teachers' perseverance and mathematical reasoning," *Infinity Journal*, vol. 12, no. 2, pp. 377-392, 2023.